education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P1

NOVEMBER 2007

MEMORANDUM

This memorandum consists of 12 pages.

QUESTION 1

1.1.1(a)	$(x+3)(x-1) = -x+1$ $x^2 + 2x - 3 = -x + 1$ $\quad(x+3)(x-1)+(x-1)=0$ $x^2 + 3x - 4 = 0$ $\quad$OR$\quad (x-1)(x+4)=0$ $(x+4)(x-1) = 0$ $\qquad\qquad x = 1$ or $x = -4$ $x = -4$ or $x = 1$	✓ multiplication / transposing ✓ equating to zero ✓ factors ✓ x-values	(4)
1.1.1(b)	$x^2 + 3x - 4 < 0$ $(x+4)(x-1) < 0$ $-4 < x < 1$	✓ factors ✓✓ answers	(3)
1.1.2	$$x^2 + 3x = 1$$ $x^2 + 3x = 1$ $\qquad\qquad x^2 + 3x + \left(\dfrac{3}{2}\right)^2 = 1 + \left(\dfrac{3}{2}\right)^2$ $x^2 + 3x - 1 = 0$ $\qquad\quad \left(x + \dfrac{3}{2}\right)^2 = \dfrac{13}{4}$ $x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(1)(-1)}}{2(1)}$ OR $x + \dfrac{3}{2} = \pm\sqrt{\dfrac{13}{4}}$ $x = \dfrac{-3 \pm \sqrt{13}}{2}$ $\qquad\qquad x = -\dfrac{3}{2} \pm \sqrt{\dfrac{13}{4}}$ $\qquad\qquad\qquad\qquad\quad x = 0,3$ or $x = -3,3$ $x = 0,3$ or $x = -3,3$	✓ standard form ✓ formula ✓ substitution ✓✓ answers	(5)
1.2	$\qquad\qquad x = 3 - y$ $\qquad\quad 2x^2 + 2y^2 = 5xy$ $\qquad 2(3-y)^2 + 2y^2 = 5(3-y)y$ $2(9 - 6y + y^2) + 2y^2 = 15y - 5y^2$ $18 - 12y + 2y^2 + 2y^2 = 15y - 5y^2$ $\qquad\quad 9y^2 - 27y + 18 = 0$ $\qquad\qquad y^2 - 3y + 2 = 0$ $\qquad\quad (y-2)(y-1) = 0$ $\qquad\quad y = 2$ or $y = 1$ $\qquad\quad x = 1$ or $x = 2$ $\qquad\qquad\qquad$**OR**	✓ making x the subject of the formula ✓ substitution ✓ multiplication ✓ simplification ✓ standard form ✓✓ y values ✓✓ x values ✓ making y the subject of the formula ✓ substitution ✓ multiplication ✓ simplification ✓ standard form	(9)

$$y = 3 - x$$
$$2x^2 + 2y^2 = 5xy$$
$$2x^2 + 2(3-x)^2 = 5(3-x)x$$
$$2x^2 + 2(9 - 6x + x^2) = 15x - 5x^2$$
$$2x^2 + 18 - 12x + 2x^2 = 15x - 5x^2$$
$$9x^2 - 27x + 18 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
$$x = 1 \text{ or } x = 2$$
$$y = 2 \text{ or } y = 1$$

✓✓y values

✓✓ x values

1.3	$f(x-1) = (x-1)^2 - 2(x-1)$	✓ substitution	
	$\qquad = x^2 - 2x + 1 - 2x + 2$	✓ multiplication completing the square	
	$\qquad = x^2 - 4x + 3$	$(✓ + 4; ✓ -1)$	
	$\qquad = (x^2 - 4x + 4) - 1$		
	$\qquad = (x-2)^2 - 1$		
	OR		
	$\quad f(x) = x^2 - 2x$		
	$\qquad = x^2 - 2x + 1 - 1$		
	$\qquad = (x-1)^2 - 1$		
	$f(x-1) = (x-1-1)^2 - 1$		
	$\qquad = (x-2)^2 - 1$		(4)

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QUESTION 2

2.1	$\sqrt[3]{125x^6} - \sqrt[4]{81x^8} + \sqrt{36x^4}$	✓ $5x^2$	
	$= 5x^2 - 3x^2 + 6x^2$	✓ $3x^2$	
	$= 8x^2$	✓ $6x^2$	
		✓ $8x^2$	(4)
2.2.1	$M = \sqrt{\dfrac{2}{2(1,5) + 5}} + \dfrac{1}{2(1,5)}$	✓ substitution	
	$M = \dfrac{1}{2} + \dfrac{1}{3}$	✓ simplification	
	$M = \dfrac{5}{6}$ is rational	✓ $M = \dfrac{5}{6}$	(3)
2.2.2	$2x + 5 > 0$	✓ $2x + 5 > 0$	
	$x > -\dfrac{5}{2}$	✓ $x > -\dfrac{5}{2}$	
	$x \neq 0$	✓ $x \neq 0$	(3)
2.3	Yes	✓ yes	
	$2^{2007}.5^{2000} = 2^7.2^{2000}.5^{2000}$	✓ $2^7.2^{2000}$ (exponential law)	
	$\qquad = 128.(2.5)^{2000}$	✓ 128	
	$\qquad = 128.(10)^{2000}$	✓ grouping bases with same exponents	
	the sum of the digits is $1 + 2 + 8 = 11$	✓ sum of digits	(5)

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QUESTION 3

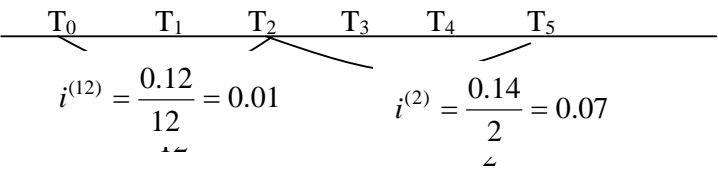| 3.1	29	✓ answer	
			(1)
3.2	$T_n = an^2 + bn + c$ From $(1 ; 1)$ $1 = a + b + c$ $c = 1 - a - b$ From $(2 ; 5)$ $5 = 4a + 2b + c$ $5 = 4a + 2b + 1 - a - b$ $4 = 3a + b$ $\qquad$ (i) From $(3; 11)$ $11 = 9a + 3b + c$ $11 = 9a + 3b + 1 - a - b$ $10 = 8a + 2b$ $\qquad$ (ii) Solving (i) and (ii) simultaneously: $\quad 8 = 6a + 2b$ $\quad 10 = 8a + 2b$ $\quad 2 = 2a$ $\quad a = 1$ $\quad b = 1$ $\quad c = -1$ $T_n = n^2 + n - 1$ OR $T_n = an^2 + bn + c$ From $(1 ; 1)$ $1 = a + b + c$ $\qquad$ (i) From $(2 ; 5)$ $5 = 4a + 2b + c$ $\qquad$ (ii) From $(3; 11)$ $11 = 9a + 3b + c$ $\qquad$ (iii) (ii) − (i) $\qquad 3a + b = 4$ $\qquad$ (iv) (iii) − (ii) $\qquad 5a + b = 6$ $\qquad$ (v) (v) − (iv) $\qquad\quad 2a = 2$ $\qquad\qquad\quad a = 1$ $\qquad\qquad\quad b = 1$ $\qquad\qquad\quad c = -1$ $\qquad\quad T_n = n^2 + n - 1$	✓ c = ✓ (i) ✓ (ii) ✓ ✓ a ✓ b ✓ c ✓✓ eqn^s (i),(ii),(iii) ✓ eqn (iv) ✓ eqn (v) ✓ a = 1 ✓ b = 1 ✓ c = −1	
			(7)
3.3	$P_n = n^2 + n - 1$ $P_{100} = 100^2 + 100 - 1 = 10\ 099$ OR $P_n = 100(101) - 1$ $P_n = 10\ 099$	✓ substitution ✓ answer ✓ substitution ✓ answer	
			(2)

[10]

QUESTION 4

4.1	$2; 1; \dfrac{1}{2}; \dfrac{1}{4}; \dfrac{1}{8}; \dfrac{1}{16}$ $\therefore$ Height reached during the 6th bounce $= \dfrac{1}{16}$	$\checkmark\checkmark$ Answer	(2)
4.2	1st bounce : $4\left(\dfrac{1}{2}\right)$ $= 2^2 . 2^{-1}$ $= 2^{2-1} = 2^1$ 2nd bounce : $4\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)$ $= 2^2 . 2^{-1} . 2^{-1}$ $= 2^{2-2} = 2^0$ 3rd bounce: $4\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = 2^2 . 2^{-1} . 2^{-1} . 2^{-1} = 2^{2-3} = 2^{-1}$ nth bounce : $4\left(\dfrac{1}{2}\right)^n = 2^{(2-n)}$ OR 1ste bounce: $2 = 2^1$ 2de bounce: $1 = 2^0$ 3de bounce: $\dfrac{1}{2} = 2^{-1}$ 4de bounce: $\dfrac{1}{4} = 2^{-2}$ . . n^{de} bounce : 2^{2-n}	$\checkmark\checkmark$ (for 1st bounce 2nd bounce and 3rd bounce) $\checkmark\checkmark$ nth bounce	(4)
4.3	$2^{2-n} = \dfrac{1}{512}$ $2^{2-n} = 2^{-9}$ $2 - n = -9$ $n = 11$ during the 11th bounce	$\checkmark$ substitution $\checkmark$ 2^{-9} $\checkmark$ equating exponents $\checkmark$ answer	(4)

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QUESTION 5

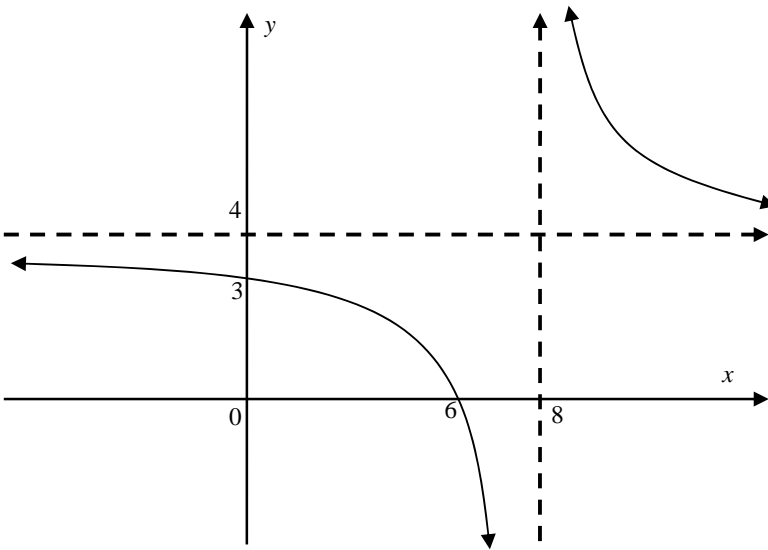| 5.1	$A = P(1-i)^n$	✓formula	
	$21500 = 86000(1-i)^4$	✓21500	
	$0.25 = (1-i)^4$	✓substitution	
	$\sqrt[4]{0.25} = 1-i$		
	$i = 1 - 0.707$		
	$i = 0.2928932188$	✓answer	
	percentage rate $= 29,29\%$	✓answer as percentage	(5)
5.2.1	$1 + i = \left(1 + \dfrac{i^{(m)}}{m}\right)^m$	✓ Formula	
	$1 + i = \left(1 + \dfrac{0,12}{12}\right)^{12}$	✓ Substitution	
	$1 + i = 1,126825$	✓Simplification	
	$i = 12,68\%$	✓ answer 12,68%	(4)
5.2.2	$\begin{array}{ccccccc} T_0 & T_1 & T_2 & T_3 & T_4 & T_5 \end{array}$		
	$i^{(12)} = \dfrac{0.12}{12} = 0.01 \qquad i^{(2)} = \dfrac{0.14}{2} = 0.07$	✓ i ✓ next i.	
	Original investment	✓formula	
	$75000 = P\left(1 + \dfrac{0,12}{12}\right)^{2\times12}\left(1 + \dfrac{0,14}{2}\right)^{3\times2}$	✓✓substitution	
		✓answer	
	$75\,000 = 1,905529326\,P$		
	$P = R\,39\,359,14$		(6)

5.3.1		✓✓ plotting of points	
		(one mark for plotting one or two correct points; two marks for plotting three or four correct points)	(2)
5.3.2	The expenditure is increasing by 0,5 billion rand each year. OR The graph is a straight line OR Linear appreciation	✓ 0,5 per year OR ✓ straight line	(1)
5.3.3	$E = 0,5t + 2$	✓2 ✓0,5t	(2)
5.3.4	$E = 0,5(8) + 2$ $E = $ R 6 billion	✓ answer	(1) **[21]**

QUESTION 6

6.1	$P(1 ; 8)$ is the turning point $f(x) = a(x-1)^2 + 8$ $f(0) = 6$ $a + 8 = 6$ $a = -2$ $f(x) = -2(x-1)^2 + 8$ $f(x) = -2(x^2 - 2x + 1) + 8$ $f(x) = -2x^2 + 4x + 6$ **OR** Shift the parabola one unit to the left Then $f(x+1) = ax^2 + 8$ $f(0) = 6$ for $x = -1$ $6 = a(-1)^2 + 8$ $a = -2$ $f(x+1) = -2x^2 + 8$ $f(x) = f((x-1)+1)$ $f(x) = -2(x-1)^2 + 8$ $f(x) = -2x^2 + 4x + 6$	✓✓ substitute Turning point ✓ substitute (0; 6) ✓ $a + 8 = 6$ ✓ $a = -2$ ✓ multiplication	
			(6)
6.2	Ave gradient $= \dfrac{f(1) - f(3)}{1-3}$ or $\dfrac{f(3)-f(1)}{3-1}$ $= \dfrac{8-0}{-2}$ $= \dfrac{0-8}{2}$ $= -4$ $= -4$	✓ $\dfrac{f(1)-f(3)}{1-3}$ ✓ substitution ✓ answer	
			(3)
6.3	$y = mx - 1$ $0 = 3m - 1$ $3m = 1$ $m = \dfrac{1}{3}$ $y = \dfrac{1}{3}x - 1$	✓ -1 ✓ substitution (3; 0) ✓ m	
			(3)
6.4	$-2x^2 + 4x + 6 = \dfrac{1}{3}x - 1$ $-6x^2 + 12x + 18 = x - 3$ $6x^2 - 11x - 21 = 0$ $(6x + 7)(x - 3) = 0$ $x = -\dfrac{7}{6}$ or $x = 3$	✓ equating ✓ standard form ✓ factors	
			(6)

	$C\left(-\dfrac{7}{6}\ ;\ -\dfrac{25}{18}\right)$	✓ x-values ✓✓ coordinate	
6.5	The parabola is reflected about the y-axis	✓ reflected ✓ y-axis	(2)
6.6	$h(x) = -2x^2 - 4x + 6$	✓✓ answer	(2) [22]

QUESTION 7

7.1	$x \in R$; $x \neq 8$	✓ answer	(1)
7.2	$f(x) = \dfrac{8}{x-8} + 4$ $0 = \dfrac{8}{x-8} + 4$ $0 = 8 + 4(x-8)$ $4x = 24$ $x = 6$	✓ $y = 0$ ✓ answer	(2)
7.3	$f(0) = \dfrac{8}{0-8} + 4$ $p = \dfrac{8}{-8} + 4$ $p = -1 + 4$ $p = 3$	✓ substitute $x = 0$ ✓ answer	(2)
7.4	$x = 8$ and $y = 4$	✓✓ answers	(2)
7.5		✓ shape ✓ y-intercept ✓ x-intercept ✓ asymptotes	(4)

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QUESTION 8
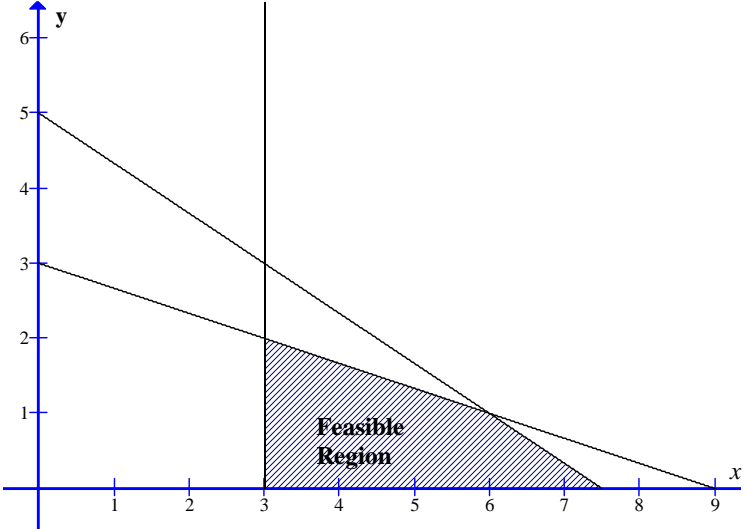
8.1	$f(x) = 1 + a.2^x$ $f(0) = 0$ $1 + a(2)^0 = 0$ $a = -1$	✓ $f(0) = 0$ ✓ substitution	(2)
8.2	$f(-15) = 1 + (-1)(2)^{-15}$ $= 1 - (2)^{-15}$ $= 0{,}99997$	✓ substitution ✓ answer	(2)
8.3	$\dfrac{1}{2} = -2^x + 1$ $\dfrac{1}{2} - 1 = -2^x$ $-\dfrac{1}{2} = -2^x$ $2^{-1} = 2^x$ $x = -1$	✓ substitution ✓ $2^{-1} = 2^x$ ✓ answer	(3)
8.4	$h(x) = f(x-2) = 1 - 2^{x-2}$	✓✓ answer	(2)
			[9]

QUESTION 9

9.1	Period $= \dfrac{360°}{2} = 180°$	✓ formula ✓ answer	(2)
9.2	$y \in [-2; 0]$	✓✓ answer	(2)
9.3	Amplitude $= \dfrac{1}{2}$	✓✓ answer	(2)

[6]

QUESTION 10

10.1	$3x + 9y \leq 27$ $4x + 6y \leq 30$ $x \geq 3$	✓✓ constraint ✓✓ constraint ✓ constraint	(5)
10.2		Sketching ✓✓ $3x + 9y \leq 27$ ✓✓ $4x + 6y \leq 30$ ✓ $x \geq 3$ ✓✓ feasible region	(8)
10.3	P = 30x + 50y	✓ coefficient of x ✓ coefficient of y	(2)
10.4	At point A (3; 2): P = 30(3) + 50(2) = 190 At point B (3; 0): P = 30(3) + 50(0) = 90 At point C (7,5; 0): P = 30(7,5) + 50(0) = 225 At point D (6; 1) : P = 30(6) + 50(1) = 230 Maximum value of P is at D (6;1) ∴ $x = 6$ i.e. 6 liters of Laughter ∴ $y = 1$ i.e. 1 liter of Joy	✓✓ substitution of points A, B, C and D ✓ x- answer ✓ y-answer	(4)
10.5	Max profit = 30(6) + 1(50) = R 230	✓✓ answer	(2)

[21]