education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

ADDITIONAL EXEMPLAR 2008

MEMORANDUM

MARKS: 150

This memorandum consists of 12 pages.

QUESTION 1

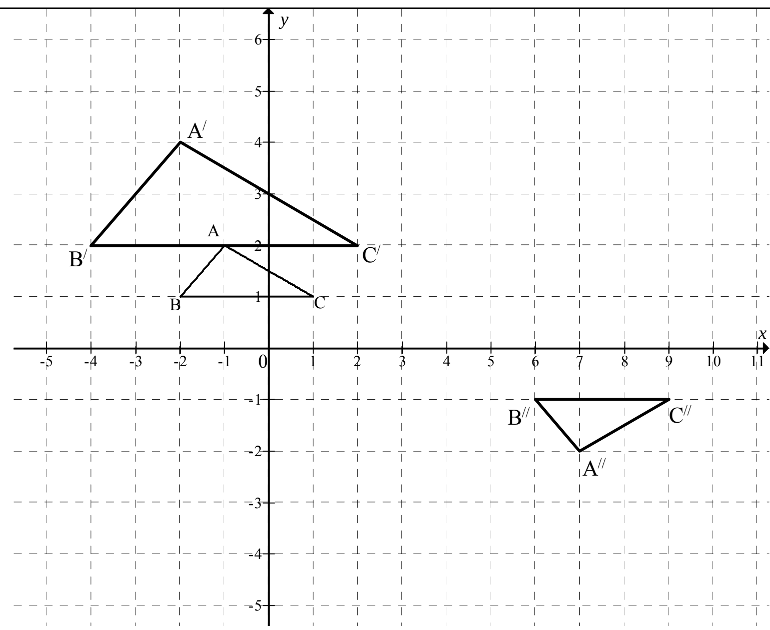
1.1	$AC = \sqrt{(-5-3)^2 + (-3-9)^2}$ $= \sqrt{64+144}$ $= \sqrt{208}$	✓ substitution ✓ simplification ✓ answer (3)
1.2	Midpoint is $\left(\frac{-5+3}{2};\frac{-3+9}{2}\right)$ M(–1 ; 3)	✓ substitution ✓ answer (2)
1.3	$m_{AC} = \frac{9+3}{3+5} = \frac{3}{2}$	✓ substitution ✓ answer (2)
1.4	$\therefore m_{BN} = -\frac{2}{3}$ $y = -\frac{2}{3}x + c$ Subst. (7 ; 2) : $2 = -\frac{2}{3}(7) + c$ $2 = \frac{-14}{3} + c$ $c = \frac{20}{3}$ $y = -\frac{2}{3}x + \frac{20}{3}$	✓ gradient of BN ✓ substitution of point ✓equation (3)
1.5	$BN = \sqrt{(7-1)^2 + (2-6)^2}$ $= \sqrt{36+16}$ $= \sqrt{52}$ Area ΔABC $= \frac{1}{2}.AC.BN$ $= \frac{1}{2}.\sqrt{208}.\sqrt{52}$ $= \frac{1}{2}\sqrt{10816}$ = 52 square units	✓ substitution ✓ answer ✓ substitution into area formula ✓answer (4)

1.6	Let α be the inclination of AC and β be the inclination of AB. $m_{AC} = \frac{3}{2}$ $\therefore \tan\alpha = 1{,}5$ $\alpha \approx 56{,}30°$ $m_{AB} = \frac{5}{12}$ $\therefore \beta \approx 22{,}61°$ $\therefore C\hat{A}B = 56{,}3° - 22{,}6° \approx 33{,}7°$ **OR** $\theta = \tan^{-1}\left(\frac{3}{2}\right) - \tan^{-1}\left(\frac{5}{12}\right)$ $\approx 33{,}7°$ **OR** $AN = \sqrt{(5-1)^2 + (-3-6)^2}$ $= \sqrt{117}$ $\tan\theta = \frac{\sqrt{52}}{\sqrt{117}}$ $\theta \approx 33{,}7°$	✓ 56,30º ✓ 22,61º ✓ $C\hat{A}B = \alpha - \beta$ ✓ 33,7º (4) **[18]**

QUESTION 2

2.1	$r^2 = (3)^2 + (-4)^2$ $= 25$ $\therefore x^2 + y^2 = 25$	✓ subst (3 ; – 4) ✓ simplification ✓ equation (3)
2.2	radius = 5 units. therefore AB = 10 units	✓ radius ✓ AB = 10 (2)
2.3	$(x-3)^2 + (y+4)^2 = 10^2$ $x^2 - 6x + 9 + y^2 + 8y + 16 = 100$ $x^2 - 6x + y^2 + 8y - 75 = 0$	✓ substitution ✓ expansion ✓ simplification (3)
2.4	A is the image of B when B is rotated through an angle of 180º about the origin.	✓ rotation ✓ 180º about the origin (2)
2.5	$m_{AB} = \frac{-4-0}{3-0}$ $= -\frac{4}{3}$	✓substitution ✓answer (2)
2.6	$m_{BC} = \frac{3}{4}$ ... tangent ⊥ radius Substitute (3 ; – 4) $-4 = \frac{3}{4}(3) + c$ $-4 = \frac{9}{4} + c$ $-16 = 9 + 4c$ $c = -\frac{25}{4}$ $\therefore y = \frac{3}{4}x - \frac{25}{4}$	✓ gradient of tangent ✓ substitution ✓ simplification ✓ value of c ✓ equation (5)
2.7	Substitute (k ; 1) into $y = \frac{3}{4}x - \frac{25}{4}$ $1 = \frac{3}{4}(k) - \frac{25}{4}$ $4 = 3k - 25$ $29 = 3k$ $k = \frac{29}{3}$	✓ substitution ✓ simplification ✓ answer (3) **[20]**

QUESTION 3

3.1.1	Reflection about the y-axis.	✓ reflection ✓ y-axis (2)
3.1.2	Translation 3 units to the left and 6 units upwards.	✓ translation ✓ 3 left and 6 upwards (2)
3.1.3	Rotation about the origin through 90º in an anticlockwise direction.	✓ rotation ✓ 90º (anticlockwise direction) (2)
3.2.1	$(x\,;y)\rightarrow(2x\,;2y)$	✓ answer (1)
3.2.2 & 3.2.6		3.2.2 ✓✓ correct coordinates (2) 3.2.6 ✓✓ correct coordinates (2)
3.2.3	$A^{/}C^{/} = 2\sqrt{5}$	✓✓ answer (2)
3.2.4	Area of $\Delta A^{/}B^{/}C^{/} = 2^2 \times$ Area of ΔABC $= 4 \times \frac{3}{2}$ $= 6$ square units	✓ $2^2 \times$ Area of ΔABC ✓ answer (2)
3.2.5	$A^{//}(-1+8\,;-2)$ $= (7\,;-2)$	✓ substitution ✓ answer (2)

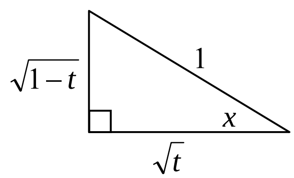
3.3.1	The coordinates for the image of C are $(x\cos(60°) - y\sin(60°)\,;\, y\cos(60°) + x\sin(60°))$ $= \left(x\left(\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right); y\left(\frac{1}{2}\right) + x\left(\frac{\sqrt{3}}{2}\right)\right)$ $= \left(\frac{x}{2} - \frac{\sqrt{3}y}{2}; \frac{y}{2} + \frac{\sqrt{3}x}{2}\right)$	✓ formula ✓ substitution ✓ ✓ special angle values ✓ simplification (5)
3.3.2	$\left(\frac{x}{2} - \frac{\sqrt{3}y}{2}; \frac{y}{2} + \frac{\sqrt{3}x}{2}\right)$ $= \left(\frac{-6}{2} - \frac{4\sqrt{3}}{2}; \frac{4}{2} - \frac{6\sqrt{3}}{2}\right)$ $= \left(-3 - 2\sqrt{3}\,;\, 2 - 3\sqrt{3}\right)$	✓✓substitution ✓answer (3) **[25]**

QUESTION 4

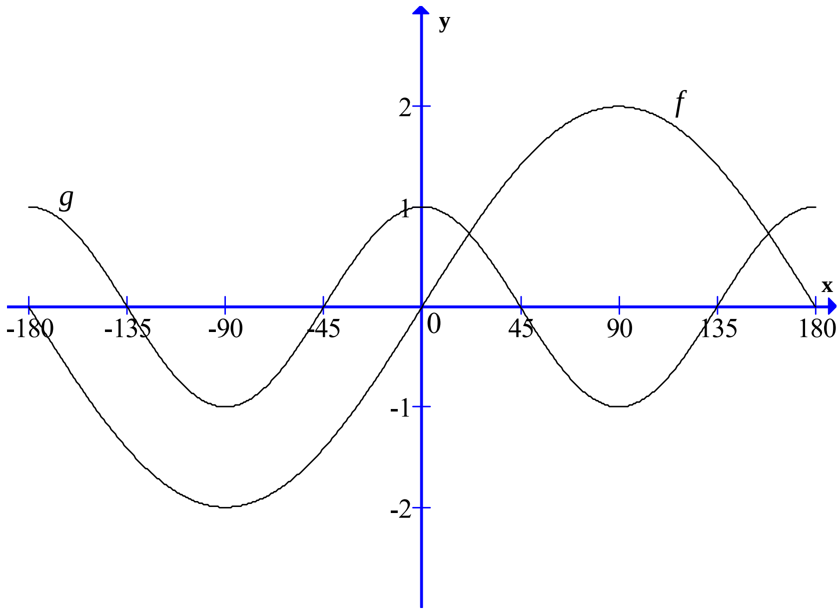
4.1.1	$\frac{(\cos 30°)(-\tan 30°)(\sin 12°)}{(-\tan 45°)(\cos 258°)}$ $= \frac{\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)(\sin 12°)}{(-1)(-\cos 78°)}$ $= \frac{\left(-\frac{1}{2}\right)(\sin 12°)}{(-1)(-\sin 12°)}$ $= -\frac{1}{2}$	✓✓ reduction ✓ special angle values ✓ - cos 78 ✓ co-ratio ✓ answer (6)
4.1.2	$\frac{\sin 2x \cos x}{2\sin x} - (-\tan x)(-\cos x)[-\sin(720° + x)]$ $= \frac{2\sin x \cos x \cos x}{2\sin x} + \left(\frac{\sin x}{\cos x}\right)(\cos x)(\sin x)$ $= \cos^2 x + \sin^2 x$ $= 1$	✓✓✓ reduction formulae ✓✓✓ identities ✓ answer (7)

4.2	$\sin 15° = \sin(45° - 30°)$ $= \sin 45° \cos 30° - \cos 45° \sin 30°$ $= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $= \frac{\sqrt{3} - 1}{2\sqrt{2}}$	✓ (45° - 30°) ✓ expansion ✓ special angle values ✓ simplification (4) **[17]**

QUESTION 5

5.1.1	$\tan x = \frac{\sqrt{1-t}}{\sqrt{t}}$ 	✓ sketch ✓ answer (2)
5.1.2	$\sin 2x = 2 \sin x \cos x$ $= 2\left(\frac{\sqrt{1-t}}{1}\right)\left(\frac{\sqrt{t}}{1}\right)$ $= 2\sqrt{t - t^2}$	✓expansion ✓✓substitution ✓answer (4)
5.2.1	$LHS = \frac{\sin x \cos x}{1 - \sin^2 x + \cos^2 x}$ $= \frac{\sin x \cos x}{\cos^2 x + \cos^2 x}$ $= \frac{\sin x \cos x}{2\cos^2 x}$ $= \frac{\sin x}{2\cos x}$ $= \frac{1}{2}\tan x$	✓ identity ✓adding terms ✓simplification ✓identity (4)
5.2.2	$\frac{1}{2}\tan x = 0$ $\tan x = 0$ $x = 0° + k.180°; k \in Z$	✓ $\frac{1}{2}\tan x = 0$ ✓simplification ✓✓answer (4) **[14]**

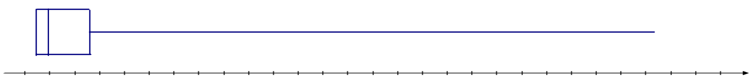
QUESTION 6

6.1		*f* ✓shape ✓intercepts ✓turning points g ✓shape ✓intercepts ✓turning points (6)
6.2	$\cos 2x = 2\sin x$ $1 - 2\sin^2 x - 2\sin x = 0$ $2\sin^2 x + 2\sin x - 1 = 0$ $\sin x = \dfrac{-2 \pm \sqrt{4 - 4(2)(-1)}}{2(2)}$ $\sin x = -1{,}366$ (n/a) or $\sin x = 0{,}366$ $\therefore x = 21.5°$ or $x = 158{,}5°$	✓identity ✓quadratic equation ✓use of quadratic formula ✓ solutions for sin x ✓✓answer for x (6)
6.3	$x = 90°$	✓answer (1) **[13]**

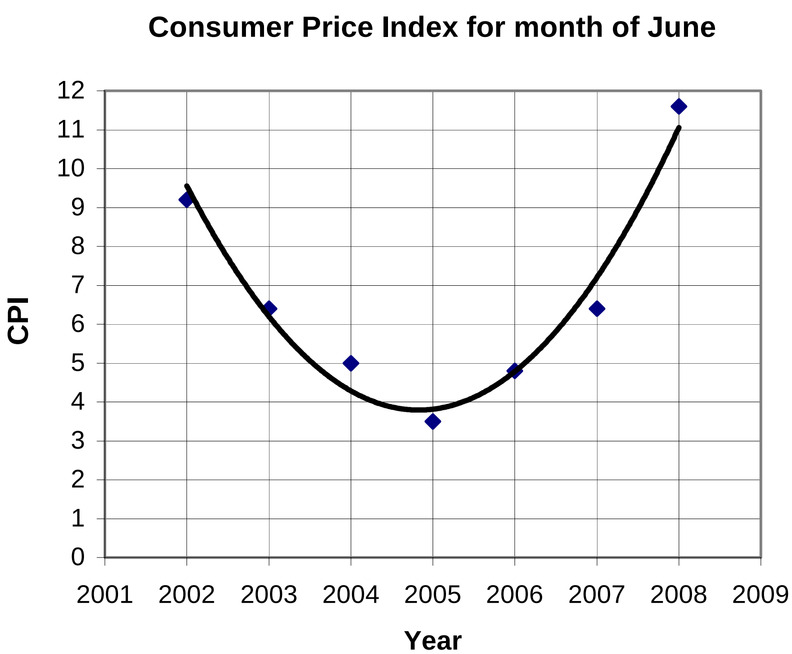
QUESTION 7

7.1	$\frac{\sin M}{e} = \frac{\sin 150^\circ}{f}$ $\sin M = \frac{e\sin 30^\circ}{f}$ $= \frac{e}{2f}$	✓ using sin rule ✓ reduction ✓special angle (3)
7.2.1	$\sin 55^\circ = \frac{50}{AC}$ $\therefore AC = \frac{50}{\sin 55^\circ}$ $AC = 61m$ $\sin 48^\circ = \frac{50}{AD}$ $\therefore AD = \frac{50}{\sin 48^\circ}$ $AD = 67{,}3m$	✓ ratio ✓answer ✓ ratio ✓answer (4)
7.2.2	$CD^2 = AC^2 + AD^2 - 2AC.AD\cos 71^\circ$ $= (61)^2 + (67{,}3)^2 - 2(61)(67{,}3)\cos 71^\circ$ $= 5577{,}18$ $CD = 74{,}68$	✓cosine rule ✓✓ substitution ✓simplification ✓answer (5)
7.2.3	Area of $\Delta ACD = \frac{1}{2}AC.AD\sin 71^\circ$ $= \frac{1}{2}(61)(67{,}3)\sin 71^\circ$ $= 1940{,}82\ m^2$	✓ area rule ✓ ✓substitution ✓ answer (4) **[16]**

QUESTION 8

8.1	Median is 2	✓ answer (1)
8.2	Upper quartile is 5 Lower quartile is 1	✓ upper quartile ✓ lower quartile (2)
8.3	Minimum value is 1 and maximum value is 51.  	✓ minimum and maximum ✓ box ✓ whisker (3)
8.4	The data is positively skewed, that is the data is skewed to the right. There is no left whisker. This implies that of the countries that won gold medals at least 25% of them won only one. The long whisker on the right shows that some countries, namely China and the USA, performed exceptionally well in the Olympics. One could say that these countries could be considered as outliers in this context.	✓ positively skewed ✓ explanation about whiskers (2) **[8]**

QUESTION 9

9.1 &	 	✓✓ plotting points ✓ labels (3)
9.2	A quadratic function would best fit this data.	✓ quadratic function ✓ line of best fit (2)

9.3	CPI for January 2008 is estimated at 9%.	✓ answer close to 9% (1) **[6]**

QUESTION 10

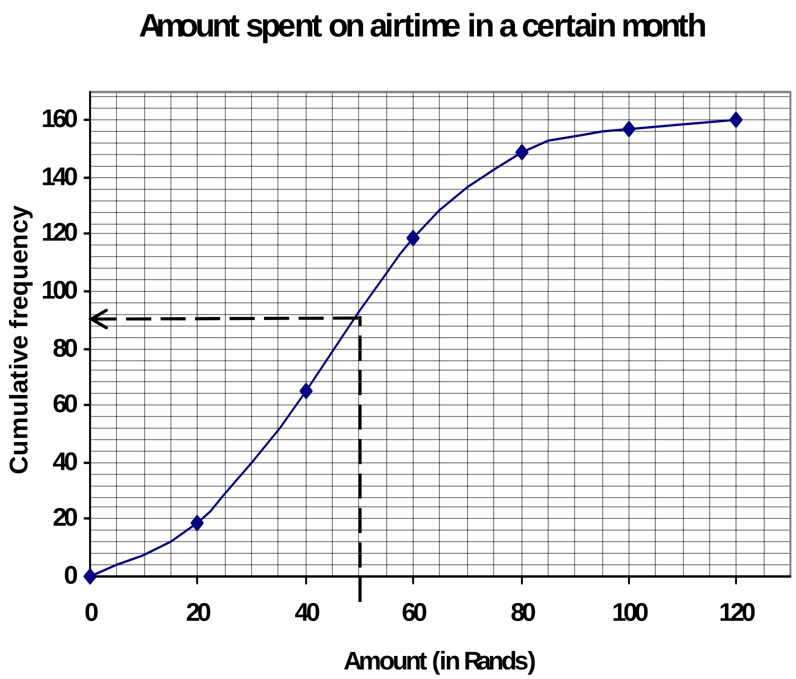
10.1	NOTE that candidates are urged to make use of available technology. By using a calculator $\sigma_n \approx 1{,}69$ (1,68518…)	✓✓✓ answer (3)
10.2	The standard deviation of 1,69 shows that there was a small variation in the maximum daily temperatures for the given period. This is confirmed by the fact that the range in the maximum temperatures is only 6ºC for the period.	✓ small variation (1) **[4]**

QUESTION 11

11.1

AMOUNT SPENT ON AIRTIME (IN RANDS)	NUMBER OF TEENAGERS	CUMULATIVE FREQUENCY
0 to less than 20	19	19
20 to less than 40	46	65
40 to less than 60	54	119
60 to less than 80	30	149
80 to less than 100	8	157
100 to less than 120	3	160

✓✓ correct totals in cumulative frequency column (2)

11.2		✓ labels ✓ points plotted correctly (2)

11.3	About 92 learners spent R50 or less on airtime.	✓ answer read off from ogive (1)
11.4		✓ midpoint column ✓ learners × midpoint column

Amount spent on airtime (in Rands)	Number of teenagers	Midpoint of interval	Teenagers × midpoint
0 to less than 20	19	10	190
20 to less than 40	46	30	1380
40 to less than 60	54	50	2700
60 to less than 80	30	70	2100
80 to less than 100	8	90	720
100 to less than 120	3	110	330
Sum			7420

$\text{Mean} = \frac{7420}{160} \approx R46{,}38$ ✓✓ mean (4)

[9]

TOTAL: 150