education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**
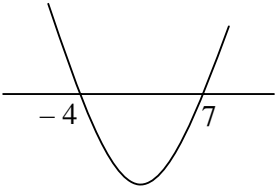
GRADE 12

MATHEMATICS P1

ADDITIONAL EXEMPLAR 2008

MEMORANDUM

MARKS: 150

This memorandum consists of 11 pages.

QUESTION 1

1.1.1	$\dfrac{1}{x}+\dfrac{5}{x-1}=6$	
	$x-1+5x=6x(x-1)$	✓ simplification ✓ standard form
	$x-1+5x=6x^2-6x$	
	$0=6x^2-12x+1$	✓ substitution ✓ simplification
	$x=\dfrac{-(-12)\pm\sqrt{(-12)^2-4(6)(1)}}{2(6)}$	
	$x=\dfrac{12\pm\sqrt{120}}{12}$	
	$x=1{,}91$ or $x=0{,}09$	✓✓ answers (6)
1.1.2	$x^2-3x\geq 28$	
	$x^2-3x-28\geq 0$	✓ standard form ✓ factors ✓ method
	$(x-7)(x+4)\geq 0$	
	OR	
	$x\leq -4$ or $x\geq 7$	
	OR	
	$x\in(-\infty;-4]\cup[7;\infty)$	✓✓ answer (5)
1.2	$2x-y=3$	
	$y=2x-3$	✓ simplification ✓ substitution
	$x^2+5x(2x-3)+(2x-3)^2=15$	
	$x^2+10x^2-15x+4x^2-12x+9-15=0$	✓ simplification
	$15x^2-27x-6=0$	
	$5x^2-9x-2=0$	✓ standard form ✓ factors ✓ answers
	$(5x+1)(x-2)=0$	
	$x=-\dfrac{1}{5}$ or $x=2$	
	$y=-\dfrac{17}{5}$ or $y=1$	✓ answers

	OR	
	$2x - y = 3$	
	$x = \dfrac{y+3}{2}$	
	$\left(\dfrac{y+3}{2}\right)^2 + 5\left(\dfrac{y+3}{2}\right)y + y^2 = 15$	
	$\dfrac{y^2 + 6y + 9}{4} + \dfrac{5y^2 + 15y}{2} + y^2 = 15$	
	$y^2 + 6y + 9 + 10y^2 + 30y + 4y^2 - 60 = 0$	
	$15y^2 + 36y - 51 = 0$	
	$5y^2 + 12y - 17 = 0$	
	$(5y + 17)(y - 1) = 0$	
	$y = -\dfrac{17}{5}$ or $y = 1$	
	$x = -\dfrac{1}{5}$ or $x = 2$	(7) **[18]**

QUESTION 2

2.1	$T_n = -5 + (n-1)(4)$ $439 = -5 + 4(n-1)$ $444 = 4(n-1)$ $n - 1 = 111$ $n = 112$ **OR** $T_n = 4n - 9$ $439 = 4n - 9$ $448 = 4n$ $112 = n$	✓ $T_n = 4n - 9$ ✓ substitution of 439 ✓ answer (3)
2.2.1	$r = \dfrac{27p^2}{81p} = \dfrac{p}{3}$	✓ ratio ✓ simplification (2)
2.2.2	$-1 < \dfrac{p}{3} < 1$ $-3 < p < 3 \quad p \neq 0$	✓ setting up inequality ✓ answer (3)

2.2.3	If $p = 2$ the sequence is $162 \; ; \; 108 \; ; \; 72 \; ; \; 48 \; ; \; ……..$ $$\therefore a = 162 \qquad ; \qquad r = \frac{2}{3}$$ $$S_\infty = \frac{a}{1 - r}$$ $$S_\infty = \frac{162}{1 - \dfrac{2}{3}}$$ $$S_\infty = 486$$	✓ value of a and r ✓ formula ✓ substitution ✓ answer (3) **[11]**

QUESTION 3

3.1	Tebogo's sequence will form a geometric sequence with common ratio 3. Thembe's sequence will form a quadratic sequence with a constant second difference 8.	✓✓ explanation (Tebogo) ✓✓ explanation (Thembe) (4)
3.2	$2 \quad\quad 6 \quad\quad 18 \quad\quad 38$ $\quad 4 \quad\quad 12 \quad\quad 20$ $2a = 8$ $a = 4$ $T_n = 4n^2 + bn + c$ $2 = 4 + b + c$ $-2 = b + c$ $6 = 4(2)^2 + 2b + c$ $-10 = 2b + c$ $b = -8$ $c = 6$ $T_n = 4n^2 - 8n + 6$ (Thembe's sequence) **OR** $2a = 8$ $a = 4$ $T_0 = 6 = c$ $T_1 = 2 = 4 + b + 6$ $b = -8$ $T_n = 4n^2 - 8n + 6$ (Thembe's sequence) Tebogo's sequence is $T_n = 2.3^{n-1}$	✓ $a = 4$ ✓✓ setting up equations ✓ value of b ✓ value of c ✓✓ answer (7)
3.3	$T_n = 4n^2 - 8n + 6$ $T_{11} = 4(11)^2 - 8(11) + 6$ $T_{11} = 402$	✓ substitution ✓ answer (2)

3.4	$S_n = \dfrac{a(r^n - 1)}{r - 1}$		
	$531440 = \dfrac{2(3^n - 1)}{3 - 1}$	✓ substitution	
	$531440 = 3^n - 1$	✓ simplification	
	$531441 = 3^n$		
	$3^{12} = 3^n$	✓ answer	
	$n = 12$		(3)
			[16]

QUESTION 4

4.1	$f(x) = -(x+1)^2 + 4$	✓ $f(x) = 0$	
	$0 = -(x+1)^2 + 4$		
	$(x+1)^2 = 4$	✓ factors	
	$x + 1 = 2 \quad or \quad x + 1 = -2$	✓✓ answer	
	$x = 1 \qquad or \quad x = -3$		(4)
	OR		
	$f(x) = -(x+1)^2 + 4$		
	$0 = -x^2 - 2x - 1 + 4$		
	$x^2 + 2x - 3 = 0$		
	$(x-1)(x+3) = 0$		
	$x = 1 \qquad or \quad x = -3$		
	A(-3 ; 0) and B(1 ; 0)		
4.2	C(0 ; 3)	✓ $q = 3$	
	$y = a.3^x + 3$	✓ substitution	
	$4 = a.3^{-1} + 3$		
	$1 = \dfrac{a}{3}$		
	$a = 3$		
	$g(x) = 3.3^x + 3$	✓ answer	(3)
4.3	$f(x) = -x^2 - 2x + 3$	✓ $f'(x) = -2x - 2$	
	$f'(x) = -2x - 2$	✓ equating to 1	
	$1 = -2x - 2$		
	$3 = -2x$		
	$x = -\dfrac{3}{2}$	✓ answer for x	
		✓ answer for y	
	Point $\left(-\dfrac{3}{2} ; \dfrac{15}{4}\right)$		(4)
4.4	$k > 4$	✓✓ answer	(2)
			[13]

QUESTION 5

5.1	$p(x) = a^x$ $8 = a^{-3}$ $8 = \dfrac{1}{a^3}$ $a^3 = \dfrac{1}{8}$ $a = \dfrac{1}{2}$	✓ substitution ✓ simplification ✓ answer (3)
5.2	$y = \log_{\frac{1}{2}} x$ **OR** $y = -\log_2 x$ **OR** $y = \log_2 \dfrac{1}{x}$	✓ answer (2)
5.3	$0 < x < 8$	✓✓ answer (2)
5.4	$q(x) = \left(\dfrac{1}{2}\right)^{x-3}$ **OR** $q(x) = 2^{-x+3}$	✓✓ answer (2) **[9]**

QUESTION 6

6.1	$y \in R - \{5\}$ **OR** $y \in (-\infty; 5) \cup (5; \infty)$	✓✓ answer (2)
6.2	$\dfrac{-3}{x+1} + 5 = -3x + 2$ $-3 + 5(x+1) = -3x(x+1) + 2(x+1)$ $-3 + 5x + 5 = -3x^2 - 3x + 2x + 2$ $3x^2 + 6x = 0$ $3x(x+2) = 0$ $x = 0$ or $x = -2$ Points of intersection are $(0 \,;\, 2)$ and $(-2 \,;\, 8)$	✓ equating ✓ simplification ✓✓ answers (4)
6.3	Reflection about the asymptote $x = -1$ **OR** Reflection about the asymptote $y = 5$ **OR** Reflection about the x-axis and translated up by 10 units	✓✓ answer (2) **[8]**

QUESTION 7

7.1	180^0	✓ answer	(1)
7.2	$x = -45^0$	✓✓ answer	(2)
7.3	$k(x) = -\sin 2x$	✓✓ answer	(2) **[5]**

QUESTION 8

8.	let $n = 2008$ $2008^2 + 2009 \times 2007 - 2006 \times 2010 - 2016 \times 2000$ $= n^2 + (n+1)(n-1) - (n-2)(n+2) - (n+8)(n-8)$ $= n^2 + n^2 - 1 - n^2 + 4 - n^2 + 64$ $= 67$ **OR** Let $n = 2000$ $2008^2 + 2009 \times 2007 - 2006 \times 2010 - 2016 \times 2000$ $= (n+8)^2 + (n+9)(n+7) - (n+6)(n+10) - n(n+16)$ $= n^2 + 16n + 64 + n^2 + 16n + 63 - (n^2 + 16n + 60) - (n^2 + 16n)$ $= 64 + 63 - 60$ $= 67$	✓ let $n = 2008$ ✓ substitution ✓ simplification ✓ answer **[4]**

QUESTION 9

9.1	$35000 = 5000\left(1 + \dfrac{0{,}096}{4}\right)^{4n}$ $7 = \left(1 + \dfrac{0{,}096}{4}\right)^{4n}$ $\log 7 = \log\left(1 + \dfrac{0{,}096}{4}\right)^{4n}$ $\log 7 = 4n \log\left(1 + \dfrac{0{,}096}{4}\right)$ $4n = 82{,}0486988...$ $n = 20{,}51$ years	✓ $4n$ ✓ substitution ✓ log both sides ✓ use of power law ✓ answer (5)

9.2.1	$192000 = \dfrac{x\left[1-\left(1+\dfrac{0,12}{12}\right)^{-60}\right]}{\dfrac{0,12}{12}}$ $1920 = x\left[1-\left(1+\dfrac{0,12}{12}\right)^{-60}\right]$ $x = $ R 4 270,93	✓ $\dfrac{0,12}{12}$ ✓ substitution ✓ 60 months ✓ answer (4)
9.2.2	Balance Outstanding $= 192000\left(1+\dfrac{0,12}{12}\right)^{45} - \dfrac{4270,93\left[\left(1+\dfrac{0,12}{12}\right)^{45}-1\right]}{\dfrac{0,12}{12}}$ $= 300\,443,66 - 241\,226,7165\ldots$ $= $ R 59 216,95	✓ $\dfrac{0,12}{12}$ ✓ 45 ✓ 4270,93 ✓✓ substitution ✓✓ answer (7) **[16]**

QUESTION 10

10.1	$f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\dfrac{(x+h)^3 - x^3}{h}$ $= \lim_{h\to 0}\dfrac{(x+h)(x^2+2xh+h^2)-x^3}{h}$ $= \lim_{h\to 0}\dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$ $= \lim_{h\to 0}\dfrac{h(3x^2+3xh+h^2)}{h}$ $= \lim_{h\to 0}(3x^2+3xh+h^2)$ $= 3x^2$	✓ substitution ✓ simplification ✓ simplification ✓ common factor ✓ answer (5)
10.2.1	$y = \dfrac{2}{5\sqrt{x}} - \sqrt[3]{x}$ $y = \dfrac{2}{5}x^{-\frac{1}{2}} - x^{\frac{1}{3}}$ $\dfrac{dy}{dx} = -\dfrac{1}{5}x^{-\frac{3}{2}} - \dfrac{1}{3}x^{-\frac{2}{3}}$ $= -\dfrac{1}{5\sqrt{x^3}} - \dfrac{1}{3\sqrt[3]{x^2}}$	✓✓ simplification (one per term) ✓✓ answer (4)

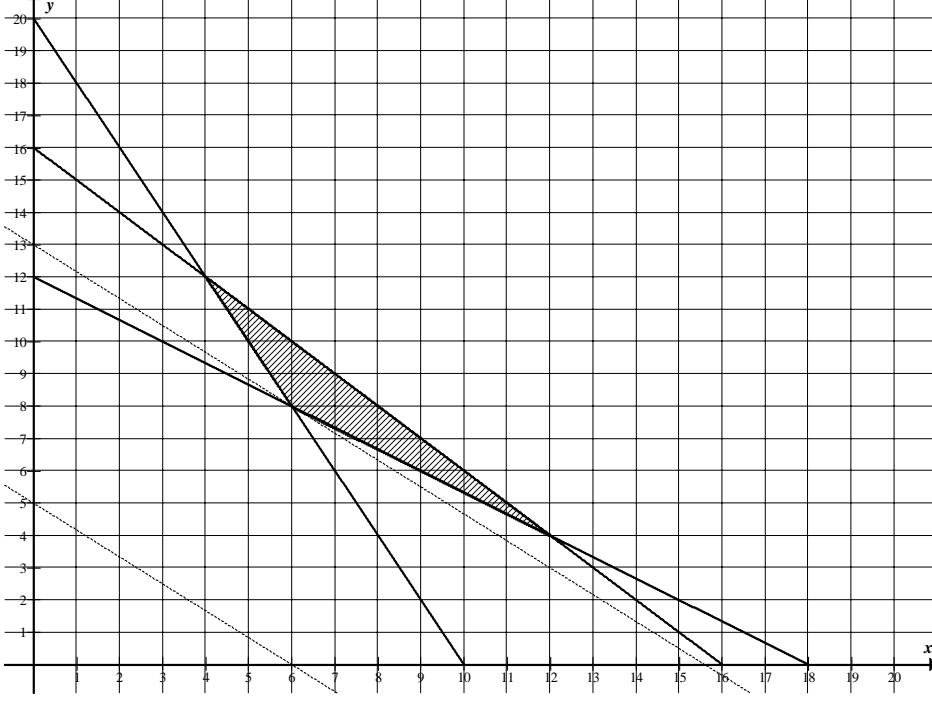| 10.2.2 | $y = \dfrac{x^4 - 3x^2 + 7}{x}$

 $y = x^3 - 3x + 7x^{-1}$

 $\dfrac{dy}{dx} = 3x^2 - 3 - 7x^{-2}$

 $\quad = 3x^2 - 3 - \dfrac{7}{x^2}$ | ✓✓ simplification

 ✓✓ answer

 (4)
 [13] |

QUESTION 11

11.1	$f(x) = x^3 + x^2 - 5x + 3$ $0 = (x-1)(x^2 + 2x - 3)$ $0 = (x-1)(x-1)(x+3)$ $x = 1$ or $x = -3$ x-intercepts are $(1\,;\,0)$ and $(-3\,;\,0)$ y-intercept is $(0\,;\,3)$	✓ $f(x) = 0$ ✓ $(x-1)$ ✓ $(x^2 + 2x - 3)$ ✓ x-intercepts ✓ y-intercept (5)
11.2	$f'(x) = 3x^2 + 2x - 5$ $0 = 3x^2 + 2x - 5$ $0 = (3x+5)(x-1)$ Turning Points are $(1\,;\,0)$ and $\left(-\dfrac{5}{3}\,;\,\dfrac{256}{27}\right)$ or $\left(-\dfrac{5}{3}\,;\,9{,}48\right)$	✓ $f'(x) = 3x^2 + 2x - 5$ ✓ $f'(x) = 0$ ✓ factors ✓ answer ✓ answer (5)
11.3	$f''(x) = 6x + 2$ $0 = 6x + 2$ $x = -\dfrac{1}{3}$	✓ $f''(x) = 6x + 2$ ✓ $f''(x) = 0$ ✓ answer (3)
11.4		✓ intercepts ✓ turning points ✓ shape (3) **[16]**

QUESTION 12

12.1	Height of the cylinder $= 2x$ By Pythagoras (radius of cylinder)$^2 = \left(5\sqrt{3}\right)^2 - x^2$ $\qquad\qquad\qquad = 75 - x^2$ $V = \pi r^2 h$ $V = \pi(75 - x^2)2x$ $V = 150\pi x - 2\pi x^3$	✓ substitution ✓ answer for r ✓ substitution (3)
12.2	$\dfrac{dV}{dx} = 150\pi - 6\pi x^2$ $0 = 150\pi - 6\pi x^2$ $x^2 = 25$ $x = \pm 5$ $x = 5 \quad x > 0$ Height of the cylinder $= 2(5)$ cm $\qquad\qquad\qquad\qquad\quad = 10$ cm	✓ $\dfrac{dV}{dx} = 150\pi - 6\pi x^2$ ✓ $\dfrac{dV}{dx} = 0$ ✓ answer for x ✓ answer (4) **[7]**

QUESTION 13

13.1	$x + y \leq 16$ $120x + 60y \geq 1200$ $2000x + 3000y \geq 36000$ $x, y \in N_0$	✓ answer ✓ answer ✓ answer ✓ answer (4)
13.2		✓✓✓ graphs of constraints ✓ feasible region (4)
13.3	$C = 40000x + 48000y$	✓ answer (1)
13.4	$m = -\dfrac{40000}{48000}$ $m = -\dfrac{5}{6}$ Minimum Cost at (6 ; 8) i.e. 6 Silver Jets and 8 Golden Flyers	✓ gradient of search line ✓✓ answer (3)
13.5	$C = 40000(6) + 48000(8)$ $\quad = R\ 624\ 000$	✓ substitution ✓ answer (2) **[14]**

TOTAL: 150