education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

ADDITIONAL EXEMPLAR 2008

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 2 diagram sheets and a 1-page formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. TWO diagram sheets for answering QUESTION 11.4 and QUESTION 13.2 are included at the end of this question paper. Write your examination number in the spaces provided on these sheets and hand them in together with the ANSWER BOOK.

7. An information sheet, with formulae, is included at the end of this question paper.

8. Number the answers correctly according to the numbering system used in this question paper.

9. It is in your own interest to write legibly and to present work neatly.

QUESTION 1

1.1 Solve for *x*, rounded off to TWO decimal places where necessary:

1.1.1 $\frac{1}{x}+\frac{5}{x-1}=6$ (6)

1.1.2 $x^2-3x\geq 28$ (5)

1.2 Solve for x and y simultaneously:

$2x-y=3$

$x^2+5xy+y^2=15$ (7)

[18]

QUESTION 2

2.1 Determine how may terms the following sequence has:

$-5\ ;\ -1\ ;\ 3\ ;\ 7\ ;\ \ldots\ ;\ 439$ (3)

2.2 Consider the following geometric sequence: $81p\ ;\ 27p^2\ ;\ 9p^3\ ;\ 3p^4\ ;\ \ldots\ (p\neq 0)$

2.2.1 Determine the common ratio of the sequence in terms of p. (2)

2.2.2 For which value of p will the sequence converge? (3)

2.2.3 Calculate S_∞ if $p=2$. (3)

[11]

QUESTION 3

Tebogo and Thembe were investigating the following sequence of numbers:

$2\ ;\ 6\ ;\ 18\ ;\ \ldots$

3.1 Tebogo claimed that the fourth term of the sequence is 54. Thembe disagreed and said that the next term is 38. Explain why it is possible that both of them are correct. (4)

3.2 Determine the general term of the sequence in both cases. (7)

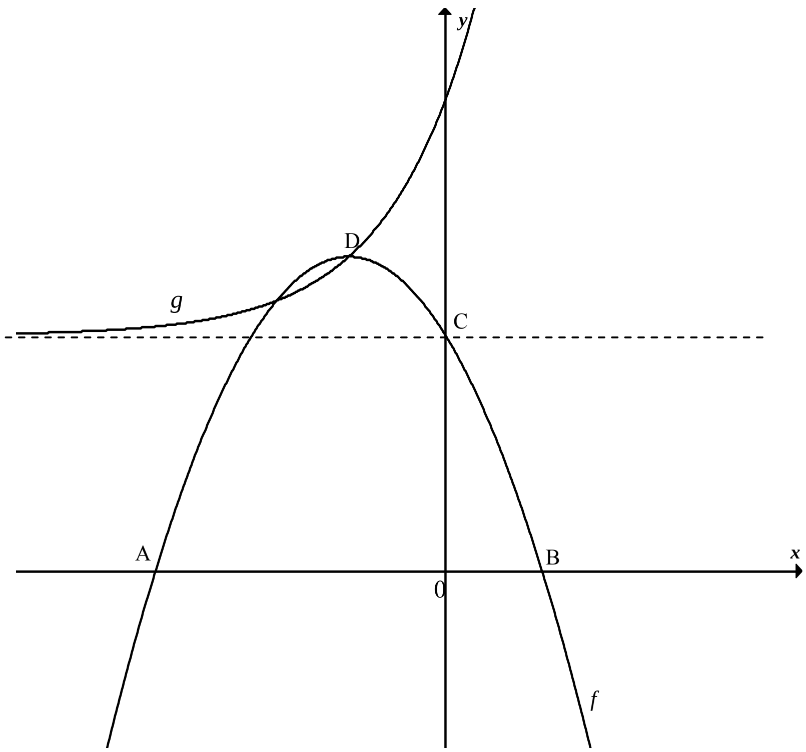
3.3 Calculate the 11^{th} term of the sequence according to Thembe's pattern. (2)

3.4 How many terms of Tebogo's pattern will give a sum of 531 440? (3)

[16]

QUESTION 4

The graphs of $f(x) = -(x+1)^2 + 4$ and $g(x) = a.3^x + q$ are sketched below.
A and B are the x-intercepts of f.
C is the y-intercept of f and lies on the asymptote of g. The two graphs intersect in D, the turning point of f.

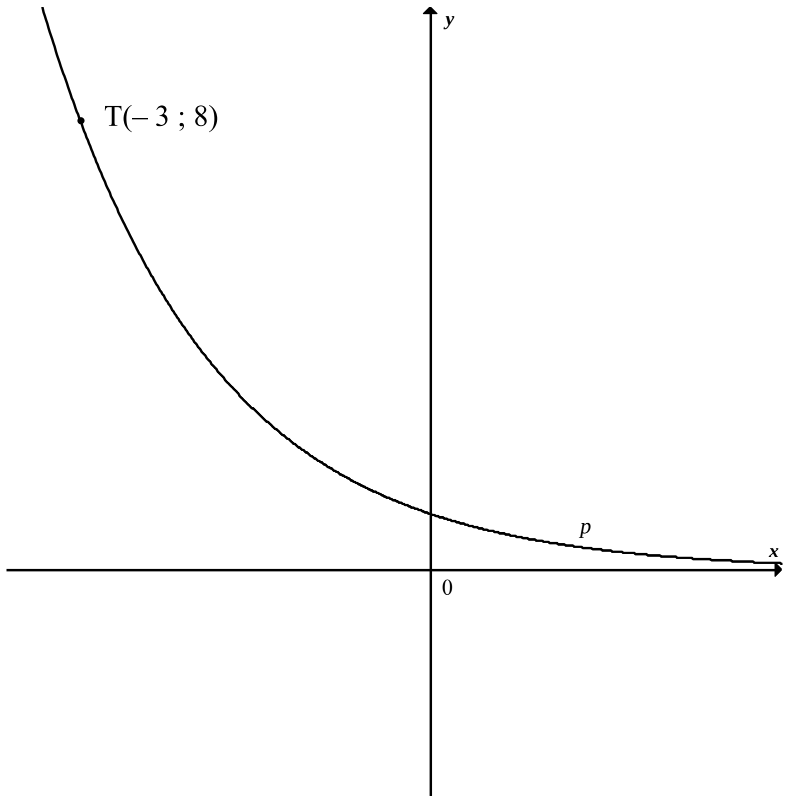


4.1	Determine the coordinates of A and B.	(4)
4.2	Determine the equation of g.	(3)
4.3	Calculate the coordinates of the point on f for which the tangent to f will have a gradient of 1.	(4)
4.4	Write down the values of k for which $f(x) - k$ will always be a negative value.	(2)

[13]

QUESTION 5

The graph of $p(x) = a^x$ is sketched below. The point T(– 3 ; 8) lies on the graph of p.



5.1 Calculate the value of a. (3)

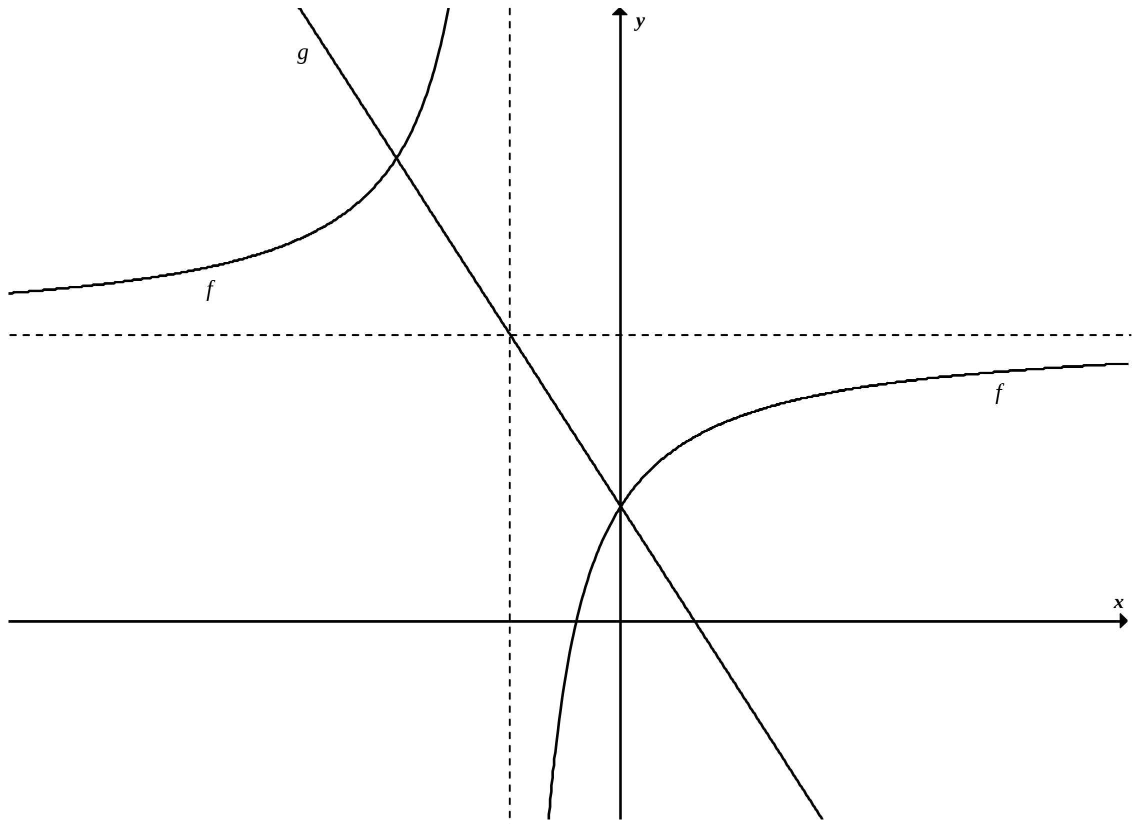
5.2 Write down the equation of $p^{-1}(x)$ in the form $y = \ldots$ (2)

5.3 For which values of x will $p^{-1}(x) > -3$? (2)

5.4 Write down the equation of q if q is the result of p shifted 3 units to the right. (2)
[9]

QUESTION 6

The graphs of $f(x) = \dfrac{-3}{x+1} + 5$ and $g(x) = -3x + 2$ are sketched below.



6.1 Write down the range of $f(x)$. (2)

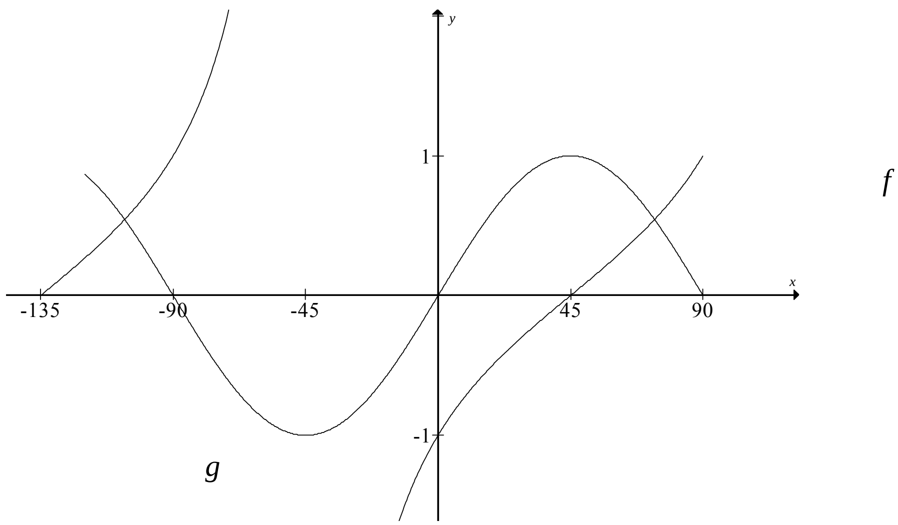
6.2 Determine the coordinates of the points of intersection of f and g. (4)

6.3 Describe the transformation of f to h if $h(x) = \dfrac{3}{x+1} + 5$ (2)

[8]

QUESTION 7

The graphs of the functions $f(x) = \tan(x - 45°)$ and $g(x) = \sin 2x$ for $x \in [-135° \,;\, 90°]$ are sketched below.



7.1 Write down the period of g. (1)

7.2 For which value(s) of x will f have an asymptote if $x \in [-135° \,;\, 90°]$? (2)

7.3 Write down the equation of k if k is the reflection of g in the x-axis. (2)
[5]

QUESTION 8

Karabo does not have a calculator and has to find the value of the following expression:

$$2008^2 + 2009 \times 2007 - 2006 \times 2010 - 2016 \times 2000$$

Show Karabo how she can calculate the answer without using a calculator. **[4]**

QUESTION 9

9.1 R5 000 is invested at 9,6% p.a., interest compounded quarterly. After how many years will the investment be worth R35 000? (5)

9.2 Waydene wants to buy a car costing R192 000. She takes out a loan for 5 years with interest charged at 12% p.a. compounded monthly.

9.2.1 Calculate the monthly instalments that Waydene will have to pay on the car loan. (4)

9.2.2 After Waydene has paid 45 instalments she decides to settle the balance on the car loan. Calculate the lump sum Waydene will need to pay after she has paid the 45^{th} instalment. (7)

[16]

QUESTION 10

10.1 Determine $f'(x)$ by first principles if $f(x) = x^3$ (5)

10.2 Use differentiation rules to differentiate the following:

10.2.1 $y = \dfrac{2}{5\sqrt{x}} - \sqrt[3]{x}$ (4)

10.2.2 $y = \dfrac{x^4 - 3x^2 + 7}{x}$ (4)

[13]

QUESTION 11

Given: $f(x) = x^3 + x^2 - 5x + 3$

11.1 Calculate the x- and y-intercepts of f. (5)

11.2 Determine the turning points of f. (5)

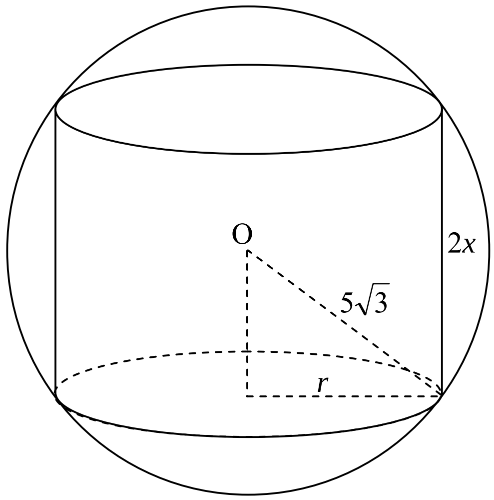
11.3 Determine the x-value of the point of inflection. (3)

11.4 Hence, sketch the graph of f on DIAGRAM SHEET 1. Show clearly ALL intercepts with the axes and any turning points. (3)

[16]

QUESTION 12

A cylinder with height $2x$ units is placed inside a sphere with radius $5\sqrt{3}$ units.
O is the centre of the sphere.



12.1 Show that the volume of the cylinder can be expressed as $V = 150\pi x - 2\pi x^3$. (3)

12.2 Calculate the height of the cylinder if it is of maximum volume. (4)
[7]

QUESTION 13

A travel agency has to transport a minimum of 1 200 passengers and 36 000 kg of luggage from one airport to another. Two types of aircraft are available. The Silver Jet can transport a maximum of 120 passengers and 2 000 kg of luggage in one flight. The Golden Flyer can transport a maximum of 60 people and 3 000 kg of luggage in one flight. The travel agency may not use more than 16 aircrafts.

Let the number of Silver Jets be x and the number of Golden Flyers be y.

13.1 Write down the constraints of the above problem. (4)

13.2 Graph the constraints on DIAGRAM SHEET 2 and clearly indicate the feasible region. (4)

13.3 The rental for a Silver Jet is R40 000 and R48 000 for a Golden Flyer per flight. Write down the cost equation for hiring the aircrafts. (1)

13.4 Use a search line to determine how many of each aircraft the travel agency must hire so that the cost is a minimum. (3)

13.5 Determine the minimum cost of transporting the passengers and the luggage. (2)
[14]

TOTAL: 150

EXAMINATION NUMBER:													

DIAGRAM SHEET 1

QUESTION 11.4

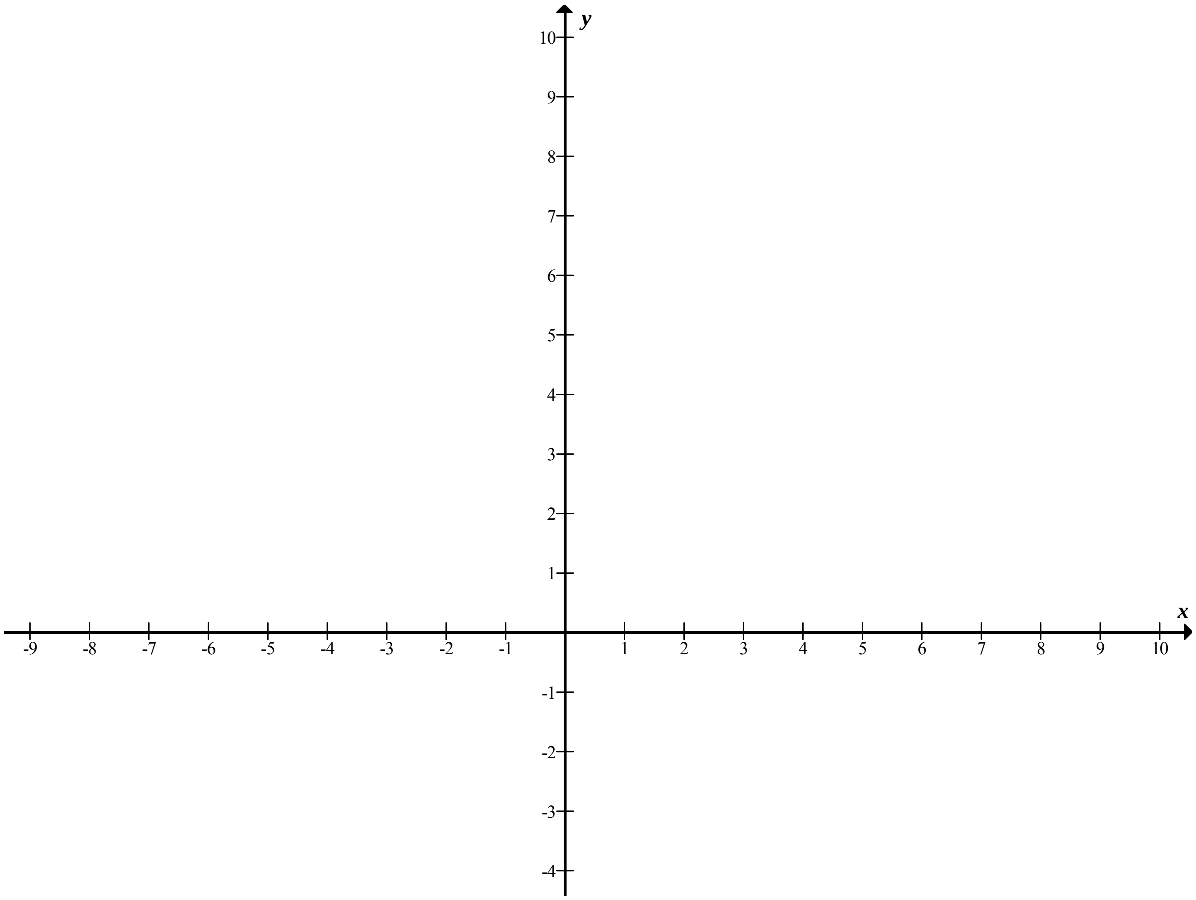


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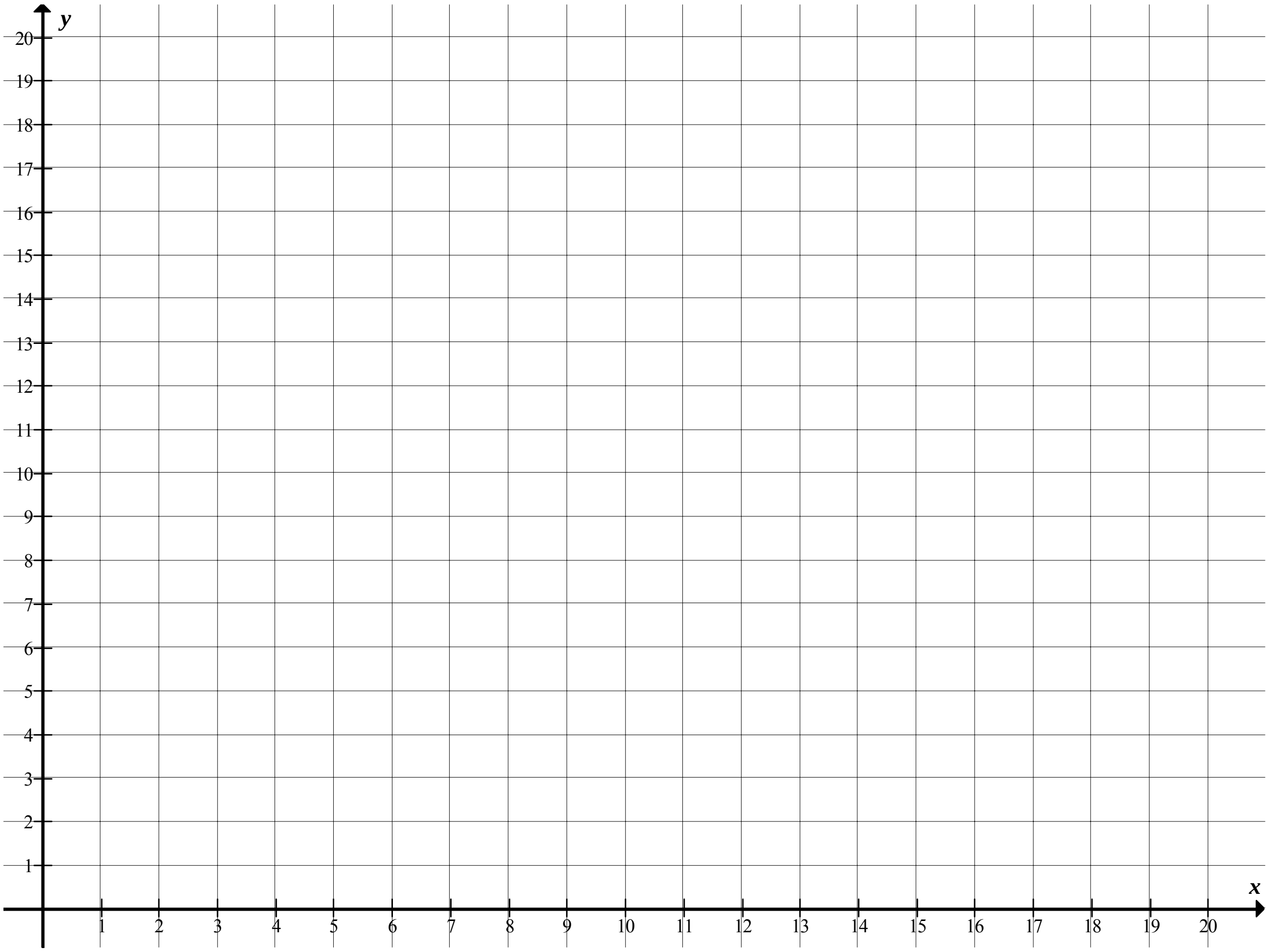
DIAGRAM SHEET 2

QUESTION 13.2



INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad \text{area } \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ and/en } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$