education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

EXEMPLAR 2008

MEMORANDUM

This memorandum consists of 12 pages.

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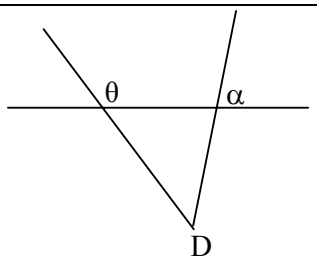
QUESTION 1

1.1	$AC = \sqrt{(2+4)^2 + (5-3)^2}$ $AC = \sqrt{40}$ $AC = 2\sqrt{10}$	✓ substitution ✓ answer (2)
1.2	$M(\frac{-4+2}{2}; \frac{3+5}{2}) \quad \therefore M(-1;4)$	✓ substitution ✓ answer (2)
1.3	$m_{BD} = \frac{10-(-2)}{-3-1} = \frac{12}{-4} = -3$ $m_{AC} = \frac{5-3}{2-(-4)} = \frac{2}{6} = \frac{1}{3}$ $\therefore m_{BD} \times m_{AC} = -3 \times \frac{1}{3} = -1$ $\therefore BD \perp AC$ $Midpoint\ BD\left(\frac{-3+1}{2}; \frac{10-2}{2}\right) = Midpoint\ of\ AC$ $= (-1\ ;\ 4)$ $\therefore$ bisect at $90°$	✓ answer ✓ answer ✓ -1 ✓ coordinates ✓ = Midpoint AC (5)
1.4	Area ΔABC $= \frac{1}{2}.AC.MB$ $= \frac{1}{2}.\sqrt{40}.\sqrt{(10-4)^2 + (-3+1)^2}$ $= \frac{1}{2}\sqrt{40}.\sqrt{40}$ $= 20$	✓ formula ✓ substitution ✓ MB = $\sqrt{40}$ ✓answer (4)
1.5	$m_{DC} = \frac{3+2}{-4-1} = -1$ $y + 2 = -1(x-1)$ $y = -x - 1$	✓ substitution ✓ answer ✓ answer (3)
1.6	$m_{DC} = -1$ $\tan\theta = -1$ $\theta = 135°$	✓ substitution ✓ answer (2)
1.7	$m_{AD} = \frac{5+2}{2-1} = 7$ $\therefore \tan\alpha = 7$ $\alpha = 81{,}9°$ $A\hat{D}C = \theta - \alpha$ $A\hat{D}C = 135° - 81{,}9° = 53{,}1°$ OR  	✓ 7 ✓ 81,9° ✓ $A\hat{D}C = \theta - \alpha$ ✓ 53,1° (4)

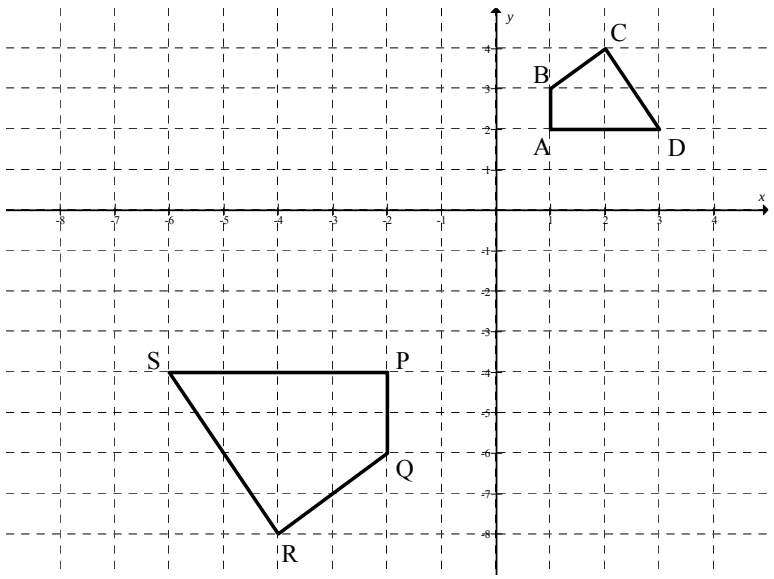
	$A\hat{D}C = 180° - (45° + 81{,}9°)$ $A\hat{D}C = 53{,}1°$ OR999 Use Cosine Rule $AC^2 = DC^2 + AD^2 - 2DC.AD\cos D$ $\therefore 40 = 50 + 50 - 2 \times 50\cos D$ $\therefore \cos D = 0.6 \quad \therefore \hat{D} = 53{,}13°$	✓ Use Cosine Rule ✓✓Substitution ✓ answer **[22]**

QUESTION 2

2.1	$m_{AB} = \frac{7-5}{12-8}$ $m_{AB} = \frac{1}{2}$ equation of AB is $y - 5 = \frac{1}{2}(x-8)$ $y = \frac{1}{2}x + 1$ $2y - x = 2$ $x - 2y + 2 = 0$	✓ $m_{AB} = \frac{1}{2}$ ✓ substitution ✓ answer (3)
2.2	At D: $x - 2(0) + 2 = 0$ $x = -2$ D(– 2 ; 0)	✓ substitution of $y = 0$ ✓ x answer (2)
2.3	$C\left(\frac{8-2}{2}; \frac{5+0}{2}\right)$ $C\left(3; \frac{5}{2}\right)$	✓ x – value ✓ y –value (2)
2.4	$AC^2 = (8-3)^2 + \left(5 - \frac{5}{2}\right)^2$ $AC^2 = 25 + \frac{25}{4}$ $AC^2 = \frac{125}{4}$ equation of the circle is $(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{125}{4}$	✓ substitution ✓ AC^2 ✓ $(x-3)$ ✓ $\left(y - \frac{5}{2}\right)$ ✓ answer (5)
2.5	gradient of tangent = – 2 (tangent ⊥ radius) equation of tangent is	✓ gradient ✓ substitution ✓ answer

2.6	$y-5=-2(x-8)$ $y-5=-2x+16$ $y=-2x+21$ Axis of symmetry $x=3$ $A^{/}(-2\ ;\ 5)$	(3) ✓ axis of symmetry ✓ x-answer ✓ y-answer (3) **[18]**

QUESTION 3

3.1.1	$P(2\ ;\ -\sqrt{3}\,)$	✓✓ coordinates (2)
3.1.2	$P(-\sqrt{3}\ ;\ 2)$	✓✓ coordinates (2)
3.2.1		✓ coordinates P ✓ coordinates Q ✓ coordinates R ✓ coordinates S (4)
3.2.2	$(x\ ;\ y)\rightarrow(-x\ ;\ -y)$ $(-x\ ;\ -y)\rightarrow(-2x\ ;\ -2y)$ $\therefore (x\ ;\ y)\rightarrow(-2x\ ;\ -2y)$	✓ $(x\ ;\ y)\rightarrow(-x\ ;\ -y)$ ✓ $(-x\ ;\ -y)\rightarrow(-2x\ ;\ -2y)$ ✓✓ $\therefore (x\ ;\ y)\rightarrow(-2x\ ;\ -2y)$ (4)
3.2.3	Area ABCD : area PQRS = 1 : 4	✓ answer (1)
3.3.1	Let r = OP = OP$^{/}$ The x co-ordinate of $P^{/}=r\cos(\alpha+30°)$ $x^{/}=r\cos(\alpha+30°)$ $=r(\cos\alpha.\cos30°-\sin\alpha.\sin30°)$ $=r\cos\alpha.\cos30°-r\sin\alpha.\sin30°$	✓ formula ✓ expansion ✓ simplification ✓ substitution

	$= r.\frac{x}{r}\cos 30° - r.\frac{y}{r}\sin 30°$ $= x.\frac{\sqrt{3}}{2} - y.\frac{1}{2}$ The y co-ordinate of P' is $r\sin(\alpha + 30°)$ Similarly $y' = y\cos 30° + x\sin 30°$ $= y.\frac{\sqrt{3}}{2} + x.\frac{1}{2}$	✓ formula ✓ expansion ✓ simplification ✓ substitution (8)

3.4	$K' = (x.\frac{\sqrt{3}}{2} - y.\frac{1}{2}\ ;\ y.\frac{\sqrt{3}}{2} + x.\frac{1}{2})$ $= (4.\frac{\sqrt{3}}{2} - 3.\frac{1}{2}; 3.\frac{\sqrt{3}}{2} + 4.\frac{1}{2})$ $= (1.96\ ;\ 4.60)$ $L' = (x.\frac{\sqrt{3}}{2} - y.\frac{1}{2}\ ;\ y.\frac{\sqrt{3}}{2} + x.\frac{1}{2})$ $= (3.\frac{\sqrt{3}}{2} - 6.\frac{1}{2}; 6.\frac{\sqrt{3}}{2} + 3.\frac{1}{2})$ $= (-0.40\ ;\ 6.70)$	✓ x-coordinate of K$'$ ✓ y-coordinate of K$'$ ✓ x-coordinate of L$'$ ✓ y-coordinate of L$'$ (4) **[25]**

QUESTION 4

4.1

$$\frac{\sin 140°.\tan(-315°)}{\cos 230°.\sin 420°}$$

$$= \frac{\sin 40°.(\ \tan 315°)}{(\ \cos 50°).\sin 60°}$$

$$= \frac{\sin 40°.\tan 45°}{\sin 40°.\sin 60°}$$

$$= \frac{-1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{-2}{\sqrt{3}}$$

✓ $\sin 40°$
✓ $-\cos 50°$
✓ $\tan 45°$
✓ $\sin 60°$
✓ $\sin 40°$
✓ answer

(6)

4.2

$$\tan(180° + x).\cos(540° + x)\left(\sin(-x) + \frac{\sin^2(90° - x)}{\cos(90° + x)}\right)$$

$$= \tan x.(-\cos x)\left(-\sin x + \frac{\cos^2 x}{-\sin x}\right)$$

$$= \frac{\sin x}{\cos x}.(-\cos x)\left(\frac{\sin^2 x + \cos^2 x}{-\sin x}\right)$$

$$= \sin^2 x + \cos^2 x$$

$$= 1$$

✓ $\tan x$
✓ $-\cos x$
✓ $-\sin x$
✓ $\cos^2 x$
✓ $-\sin x$
✓ $\tan x = \frac{\sin x}{\cos x}$
✓ $\sin^2 x + \cos^2 x$
✓ answer

(8)

4.3

$$\sin 15°$$

$$= \sin(45° - 30°)$$

$$= \sin 45°.\cos 30° - \cos 45°.\sin 30°$$

$$= \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}.\frac{1}{2}$$

$$= \frac{\sqrt{2}\left(\sqrt{3} - 1\right)}{4}$$

✓ $\sin 15°$
✓ $\sin(45° - 30°)$
✓ expansion
✓✓ substitution

(5)

4.4.1

$$\cos 2\theta + 3\cos\theta - 1$$

$$= 2\cos^2\theta - 1 + 3\cos\theta - 1$$

$$= 2\cos^2\theta + 3\cos\theta - 2$$

(note using 60° and 45° will also give same answer)

✓ $\cos 2\theta = 2\cos^2\theta - 1$
✓ simplification

(2)

4.4.2	$\cos 2\theta + 3\cos\theta - 1 = 0$ $2\cos^2\theta + 3\cos\theta - 2 = 0$ $(2\cos\theta - 1)(\cos\theta + 2) = 0$ $\cos\theta = \frac{1}{2}$ or $\cos\theta = -2$ invalid $\theta = \pm 60° + k.360° \quad k \in Z$	✓ factors ✓ $\cos\theta = -2$ invalid ✓ answers ✓ $k \in Z$ (4) **[25]**

QUESTION 5

5.1	Joyce did not use the correct expansion for sin(A + B).	✓ answer (1)
5.2	$\sin(45° + 21°)$ $= \sin 45°.\cos 21° + \sin 21°.\cos 45°$ $= \frac{\sqrt{2}}{2}.(t) + \sqrt{1-t^2}(\frac{\sqrt{2}}{2})$ $= \frac{\sqrt{2}}{2}\left(t + \sqrt{1-t^2}\right)$	✓ expansion ✓ substitution ✓ $\sin 21 = \sqrt{1-t^2}$ ✓ answer (4) **[5]**

QUESTION 6

6.1	$\frac{BC}{OB} = \cos\theta$ $BC = 2a.\cos\theta$	✓ $\frac{BC}{OB} = \cos\theta$ (1)
6.2	area $\Delta OBC = \frac{1}{2}.OB.BC.\sin\theta$ area $\Delta OBC = \frac{1}{2}.(2a).(2a.\cos\theta).\sin\theta$ area $\Delta OBC = a^2.2\cos\theta.\sin\theta$ area $\Delta OBC = a^2.\sin 2\theta$	✓ area rule ✓ substitution ✓ double angle (3)
6.3	Area will be a maximum when $\sin 2\theta = 1$. $\theta = 45°$	✓ answer

	ΔOBC will be isosceles C(a ; a)	(1) **[5]**

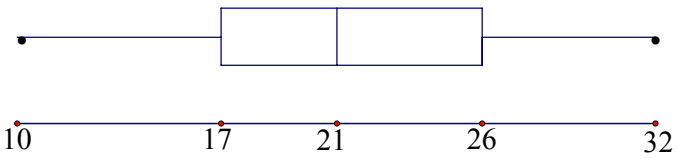
QUESTION 7

7.1	$\frac{20,7}{\text{BD}} = \cos 43,6°$ $\text{BD} = \frac{20,7}{\cos 43,6}$ BD = 28,6 m	✓ ratio ✓ solving for BD ✓ answer (3)
7.2	$BE^2 = (28,58)^2 + (28,1)^2 - 2(28,58)(28,1)\cos 35,7°$ $= 302,0610874$ … BE = 17,4 m	✓✓ cos rule ✓ substitution ✓ answer (4)
7.3	Area ΔBEC $= \frac{1}{2}(20,7)(17,4)\sin 63°$ = 160,4 square metres	✓ substitution ✓ answer (2) **[9]**

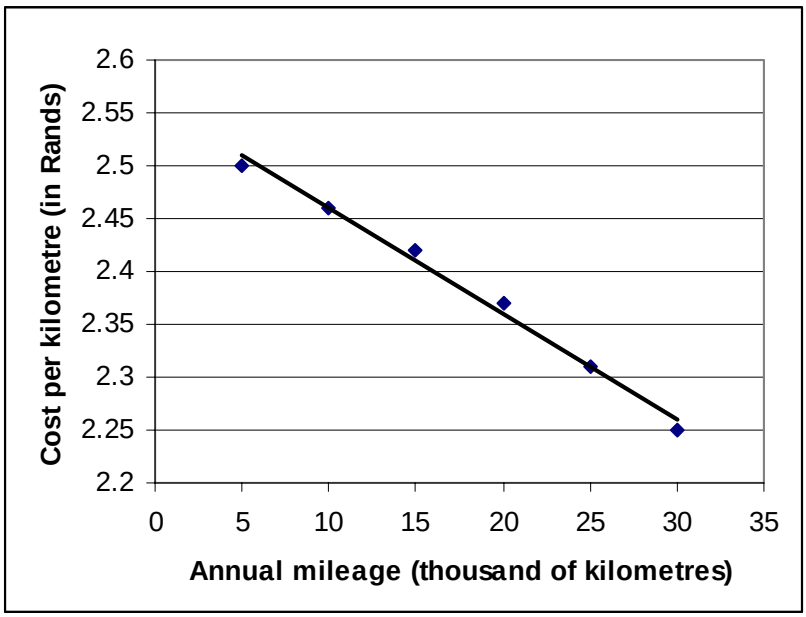
QUESTION 8

8.1	$\cos\frac{x}{2} = \sin(x-30°)$ $\cos\frac{x}{2} = \cos(90° - x + 30°)$	✓cos(90° – x + 30°)
	$\frac{x}{2} = 120° - x + k.360°$ or $\frac{x}{2} = -120° + x + k.360°$ $k \in Z$	✓✓ answer
	$\frac{3x}{2} = 120° + k.360°$ $\frac{-x}{2} = -120° + k.360°$	
	$x = 80° + k.240°$ $x = 240° - k.720°$	✓✓ answers
	$x = 80°; -160°$	✓✓ answer
	A(-160º ; 0,174) and B(80º ; 0,766)	(7)
8.2	$-160° < x < 80°$	✓ critical values ✓ statement
		(2)
		[9]

QUESTION 9

9.1	10, 13, 13, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 30, 32. Median = 21	✓ ordered data ✓ answer
		(2)
9.2	lower quartile = 17 upper quartile = 26	✓ Q_1 ✓ Q_3
		(2)
9.3		✓✓quartiles ✓ ✓box ✓ whiskers
		(5)
9.4	The range of distances that Geoff and Thabo's travelled is the same. Thabo's summary is skewed to the right while Geoff's summary is more evenly distributed. This suggest that Thabo had covered greater distances than Geoff. Also Thabo's median(viz. 25) is larger than Geoff's median(21).	✓ range the same ✓ explanation of skewed data.
		(2)
		[11]

QUESTION 10

10.1 & 10.2		✓✓ plotting points ✓ labels (3) ✓✓ line of best fit (2)
10.3	R 2,47 or R 2,48	✓ answer in this range (1) **[6]**

QUESTION 11

11.1

Percentages	Midpoint of interval (x)	Frequency (f)	Total ($f \times x$)
10 – 19	14,5	6	87
20 – 29	24,5	14	343
30 – 39	34,5	16	552
40 – 49	44,5	11	489,5
50 – 59	54,5	3	163,5
Sum			1635

✓midpoints
✓✓✓total)one mark for every two numbers correct in the last column)

$$\text{Mean} = \frac{1635}{50} = 32,7$$

✓ answer

(5)

11.2

$$\bar{x} = \frac{\sum x}{n} = \frac{7+4+9+4+9+5+4+6}{8} = 6$$

✓✓ mean

$$\delta = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

✓ formula

$$= \sqrt{\frac{(7-6)^2+(4-6)^2+(9-6)^2+(4-6)^2+(9-6)^2+(5-6)^2+(4-6)^2+(6-6)^2}{8}}$$

✓✓ calculation

$$= \sqrt{\frac{1+4+9+4+9+1+4+0}{8}} = \sqrt{4} = 2$$

✓ simplification

✓ answer

(7)

[12]

QUESTION 12

12.1	Supplier B	✓ answer (1)
12.2	I would select supplier A. The graph for supplier A shows a fairly consistent lifetime of their bulbs. Whilst the graph for supplier B shows that their bulbs last longer than supplier A, we also have a situation where their bulbs have a shorter lifetime than supplier A.	✓ supplier A ✓ explanation (2) **[3]**