education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P3

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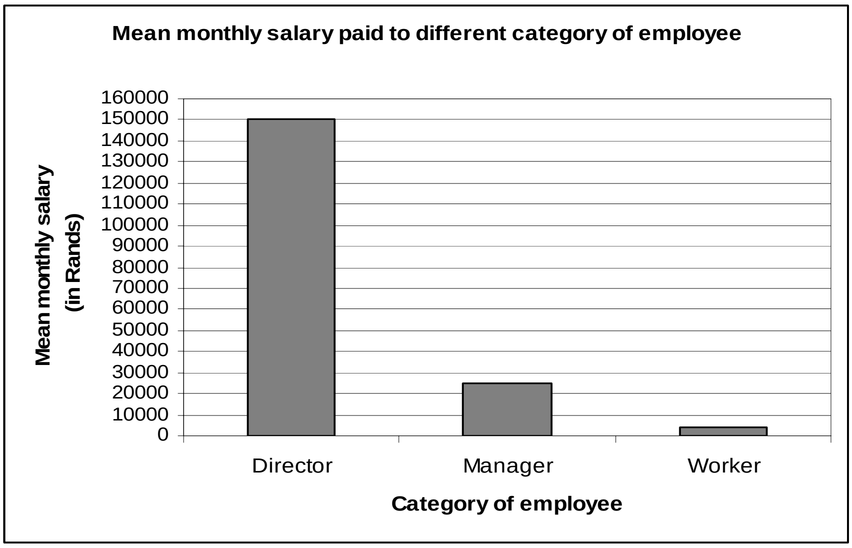
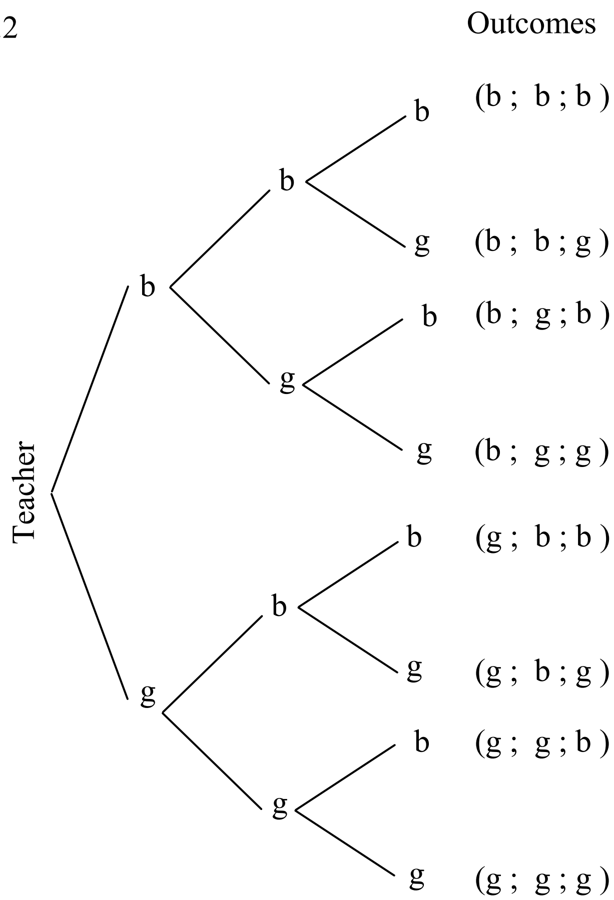
MEMORANDUM

MARKS: 100

This memorandum consists of 11 pages.

QUESTION 1	
1.1 33	✓ 33 (1)
1.2 $T_{n+1} = T_n + 5$ and $T_1 = 3$	✓✓ formula in terms of *n* ✓ $T_1 = 3$ (3) **[4]**
QUESTION 2	
2.1 Yes. According to the bar chart, the total amount spent on workers salaries is R800 000 and the total amount spent on managers salaries is R400 000, hence twice the amount.	✓ yes ✓ reason (2)
2.2. No. The total amount paid to workers is the highest of the three categories. This is expected taking into account the high number of workers in this company. The bar chart does not show the salary of individual workers and individual managers. If individual salaries are taken into account, a low percentage increase for workers will not reduce the gap in salaries. To the contrary, it will widen the gap.	✓ no ✓ reason (2)
2.3 Mean = $\frac{\text{R } 800\,000}{200} = \text{R} 4\,000$	✓ R800 000 ✓ answer (2)
2.4 	✓labels and heading ✓✓✓height of bars (4) **[10]**

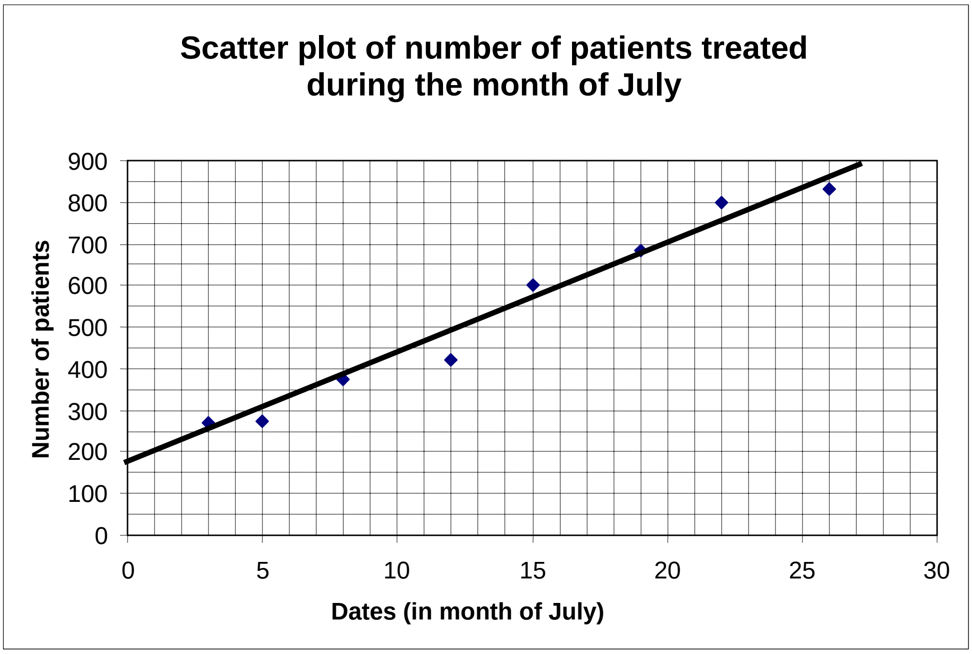
QUESTION 3	
3.1 176 – 30 = 146 and 176 + 30 = 206. Therefore the interval between 146 seconds and 206 seconds lies between one standard deviation of the mean. For the normal distribution, approximately 68% of the data lies between one standard deviation of the mean.	✓calculation ✓one standard deviation ✓68% (3)
3.2 The middle 96% of the data for a normal distribution lies between 2 standard deviations on either side of the mean. The lower limit will be 176 – 2(30) = 116 seconds. The upper limit will be 176 + 2(30) = 236 seconds. The middle 96% of the calls will be between 116 and 236 seconds.	✓ 2 standard deviations ✓ lower limit ✓ upper limit (3)
3.3 Approximately 34% of the calls are between 146 and 176 seconds. Another 50% of the calls are in excess of 176 seconds. Therefore, in total, approximately 84% of the calls are in excess of 146 seconds.	✓ 34% & 50% ✓ 84% (2) **[8]**

QUESTION 4	
4.1.1 Number of different ways in which these posts can be filled $= 3 \times 4 \times 2 = 24$.	✓✓multiplication rule ✓answer (3)
4.1.2 The post of clerk can only be filled by one person. The number of different ways in which these three posts can be filled $= 1 \times 4 \times 2 = 8$.	✓one choice for clerk ✓answer (2)
4.2.1 P(boy chosen first) $= \frac{20}{35} = \frac{4}{7} = 0{,}57$.	✓ $\frac{20}{35}$ ✓answer (1)
4.2.2 	✓ ✓ tree diagram ✓ ✓ outcomes (4)
4.2.3 $P(b\ ;\ g\ ;\ b) = \frac{20}{35} \times \frac{15}{34} \times \frac{19}{33} = \frac{190}{1309} = 0{,}15$	✓ ✓ probabilities (without replacement) ✓answer (3)
4.2.4 $P(g\ ;\ g\ ;\ g) = \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} = \frac{13}{187} = 0{,}07$	✓ probabilities (without replacement) ✓answer (2)
4.2.5 P(at least one boy) = 1 – P(three girls chosen) = 1 – 0,07 = 0,93	✓✓complementary rule ✓answer (3)

4.3 Since the teams work on the problem independently, the probabilities that both teams will solve the problem $= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = 0{,}17$. Now P(problem will be solved) = $\frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3} = 0{,}67$	✓ independent events. ✓ probability both teams solve the problem ✓ probability rule ✓answer (4) **[22]**

QUESTION 5

5.1 & 5.3



✓✓plotting points
✓labels
(3)

✓✓line of least squares (5.3)
(2)

5.2 By using a calculator : $a = 161{,}24$ (161,2371188…)
$b = 26{,}88$ (26,88275499…)
$\therefore$ equation of line of least squares is $y = 161{,}24 + 26{,}88x$

✓✓calculating the value of *a*
✓✓calculating the value of *b*
(4)

NOTE: According to the National Curriculum Statement the solutions to data-handling problems should be done with the use of a calculator. The alternative to the calculator is to use the pen-and-paper method as indicated below. All answers have been rounded to two decimal places for ease of calculations.

ALTERNATIVE

	x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
	3	270	-10,75	-260,88	2804,46	115,56	68058,37
	5	275	-8,75	-255,88	2238,95	76,56	65474,57
	8	376	-5,75	-154,88	890,56	33,06	23987,81
	12	420	-1,75	-110,88	194,04	3,06	12294,37
	15	602	1,25	71,13	88,91	1,56	5059,48
	19	684	5,25	153,13	803,93	27,56	23448,80
	22	800	8,25	269,13	2220,32	68,06	72430,96
	26	820	12,25	289,13	3541,84	150,06	83596,16
Sum	110	4247	0	0	12783,01	475,48	354350,52
Mean	13,75	530,875					

Consider the equation of the least squares line to be $\hat{y} = a + bx$

$$b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{12783,01}{475,48} = 26,88$$

(26,88443257)

✓✓calculating the value of b

Using $\hat{y} = a + bx$ and $\bar{x}$ and $\bar{y}$,
530,875 = a + (26,88443257)(13,75)
a = 161,21 (161,2140522)

✓✓calculating the value of a

Therefore equation of line of least squares is $y = 161,21 + 26,88x$ (4)

5.4 On 30 June, x = 0.
Therefore approximately 161 patients were treated on 30 June.

✓substituting 0
✓answer

ALTERNATIVE USING TABLE VALUES:
On 30 June, x = 0.
Therefore approximately 161 patients were treated on 30 June. (2)

5.5 On 24 July, x = 24.
$\hat{y} = 161,24 + 26,88(24)$ = 806,36
Approximately 806 patients were treated as at 24 July.

✓substituting 24
✓answer

ALTERNATIVE USING TABLE VALUES:

$y = 161,21 + (26,88)(24)$
$= 806,33$
Approximately 806 patients were treated as at 24 July. (2)

5.6 By using a calculator, $r = 0{,}98$ (0,9847864966...) There is a very strong positive correlation between the number of days elapsed in July and the number of patients that were treated. This would suggest that there was a rapid spread of the influenza virus in the community. **ALTERNATIVE USING TABLE VALUES:** $s_y = \sqrt{\frac{\sum(y-\bar{y})^2}{n}} = \sqrt{\frac{354350{,}52}{8}} = 210{,}46$ $s_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{475{,}48}{8}} = 7{,}71$ Using $b = r\frac{s_y}{s_x}$, we have $26{,}88 = r\frac{210{,}46}{7{,}71}$ $r = 0{,}98$ There is a very strong positive correlation between the number of days elapsed in July and the number of patients that were treated. This would suggest that there was a rapid spread of the influenza virus in the community.	✓✓calculating the value of r ✓interpretation (3) **[16]**

QUESTION 6	
6.1 equal to twice the angle at the circumference.	✓ answer (1)
6.2.1	
$\hat{T} = \hat{R}_1 = y$ .....(PR = PT)	✓ PR = PT
Now $\hat{P}_1 = 2y$ .... (ext ∠ of triangle ....)	✓ $\hat{P}_1 = 2y$
and $\hat{O}_1 = 2\hat{P}_1$ .....(angle at centre ......)	
i.e. $x = 2(2y) = 4y$	✓ answer (3)
6.2.2 (a) From 6.2.1	
$x = 4y = 120°$	✓ ✓answer
$\therefore y = 30°$	(2)
6.2.2 (b) Join Q to R and let $Q\hat{R}O = \hat{R}_3$	
$\hat{T} = y = 30°$	
but $T\hat{Q}R = T\hat{R}Q$ ..... (TQ = TR, isosceles triangle)	
$T\hat{Q}R = T\hat{R}Q = \frac{180° - 30°}{2} = 75°$	✓ calculation
Now $\hat{R}_1 + \hat{R}_2 + \hat{R}_3 = 75°$	
i.e. $30° + \hat{R}_2 + 30° = 75°$	✓ substitution
$\therefore \hat{R}_2 = 15°$	✓ answer (3) **[9]**

QUESTION 7	
7.1	
$\hat{P}_1 = \hat{Q}_1 = x$ ..... (given)	✓ $\hat{P}_1 = \hat{Q}_1 = x$
and $\hat{P}_1 = \hat{R} = x$ ....(tan - chord theorem)	✓ $\hat{P}_1 = \hat{R} = x$ ✓ reason
Now $\hat{Q}_1 = \hat{R} = x$	
$\therefore$ TQ ∥ SR ...... (corresponding angles are equal)	✓ reason (4)
7.2	
$\hat{P}_1 = \hat{S}_1 = x$ ....(TS = SP, tangents from a common point)	✓ $\hat{P}_1 = \hat{S}_1 = x$ ✓ reason
$\therefore \hat{Q}_1 = \hat{S}_1$ ....(both $= x$)	✓ conclusion
But these are angles subtended by the same line segment TP	
$\therefore$ QPTS is a cyclic quadrilateral	✓ reason (4)
7.3	
$\hat{P}_1 = \hat{Q}_1 = x$ ......(given)	
$\hat{P}_1 = \hat{Q}_2 = x$ ....(QPTS is a cyclic quad - angles subtended by same chord.)	✓ $\hat{P}_1 = \hat{Q}_2 = x$ ✓ reason
$\therefore \hat{Q}_1 = \hat{Q}_2$	
$\therefore$ TQ bisects $S\hat{Q}P$.	✓ conclusion (3) **[11]**

QUESTION 8	
8.1 In Δ ABQ, $\frac{BR}{RA} = \frac{BT}{TQ}$ ..... (RT || AQ, proportional intercept theorem) $\frac{1}{2} = \frac{k}{TQ}$ $\therefore TQ = 2k$	✓ statement & reason ✓ $\frac{1}{2} = \frac{k}{TQ}$ ✓ answer (3)
8.2.1 In Δ CRT, $\frac{CP}{PR} = \frac{5k}{2k}$ .... (RT || AQ, proportional intercept theorem) $\therefore \frac{CP}{PR} = \frac{5}{2}$	✓ ratio ✓ reason ✓ answer (3)
8.2.2 $\frac{\text{Area } \Delta \text{ RCT}}{\text{Area } \Delta \text{ ABC}} = \frac{\text{Area } \Delta \text{ RCT}}{\text{Area } \Delta \text{ BRC}} \times \frac{\text{Area } \Delta \text{ BRC}}{\text{Area } \Delta \text{ ABC}}$ .. (the ratio of the areas of triangles having equal altitude ….) $= \frac{7}{8} \times \frac{1}{3}$ $= \frac{7}{24}$	✓ ratio of areas ✓ $\frac{7}{8}$ ✓ $\frac{1}{3}$ ✓ $\frac{7}{24}$ (4) **[10]**

QUESTION 9	
9.1 In Δ BPE and Δ BDA	
$\hat{B}_1$ is common	✓ $\hat{B}_1$ is common
$\hat{P}_2 = \hat{D} = 90°$ ......(given perpendicular, ∠ in a semi - circle)	✓ $\hat{P}_2 = \hat{D} = 90°$
$B\hat{A}D = \hat{E}_3$ .... (remaining angles)	✓ $B\hat{A}D = \hat{E}_3$
$\therefore \Delta BPE /// \Delta BDA$ ..... (equiangular)	(3)
9.2 $\Delta BPE /// \Delta BDA$ .... (from 9.1)	✓ similar triangles
$\therefore \frac{BP}{BD} = \frac{PE}{DA}$ ...... (sides in proportion)	✓ reason (2)
9.3 $AB = \frac{BD.BE}{BP}$	✓ changing the subject of the formula
$AB^2 = \frac{BD^2.BE^2}{BP^2}$	✓ squaring
In Δ PBE; $BE^2 = BP^2 + PE^2$ .... (Theorem of Pythagoras)	✓ $PB^2 = BE^2 - PE^2$
$AB^2 = \frac{BD^2.(BP^2 + PE^2)}{BP^2}$	✓substitution
$AB^2 = \frac{BD^2.BP^2}{BP^2} + \frac{BD^2.PE^2}{BP^2}$	✓simplification
$AB^2 = BD^2 + \frac{BD^2.PE^2}{BP^2}$	(5) **[10]**

TOTAL: 100