education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2009(1)

MEMORANDUM

MARKS: 150

This memorandum consists of 25 pages.

- Consistent Accuracy will apply as a general rule.
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QUESTION 1

1.1	Mean = $\frac{522,5}{12} = 43,5$ **ANSWER ONLY:** Full marks	✓ 522,5 ✓ answer (2) No penalty for Rounding: Accept 43,54 ; 44
1.2	Ordered Data 9,3 14,9 15 23,6 26,1 28 32,5 60,9 65,7 71,9 76,4 98,2 Median = $\frac{28+32,5}{2}$ = 30,3 Lower quartile = $\frac{15+23,6}{2}$ =19,3 Upper quartile = $\frac{65,7+71,9}{2}$ = 68,8 The five number summary is (9,3 ; 19,3 ; 30,25 ; 68,8 ; 98,2) OR If they use the formula: Ordered Data 9,3 14,9 15 23,6 26,1 28 32,5 60,9 65,7 71,9 76,4 98,2 Position of median: $P_{50} = \frac{12+1}{2} = 6,5$ $\therefore Q_2 = \frac{28+32,5}{2} = 30,3$ Position of lower quartile: $P_{25} = \frac{13}{4}$ $\therefore Q_1 = 15 + (0,25(23,6-15)) = 17.15$ Position of upper quartile: $P_{75} = 0,75(13) = 9,75$ $\therefore Q_3 = 65,7 + (0,75(71,9-65,7)) = 70,35$ Min = 9,3 Max = 98,2 Accept any one of these five number summaries: (9,3 ; 19,3 ; 30,3 ; 68,8 ; 98,2) (9,3 ; 15 ; 30,3 ; 71,9 ; 98,2) (9,3 ; 17,2 ; 30,3 ; 70,4 ; 98,2)	✓ 9,3 ✓ 19,3 ✓ 30,3 ✓ 68,8 ✓ 98,2 (5) If indicated on the box and whisker diagram in 1.3 – 5 marks

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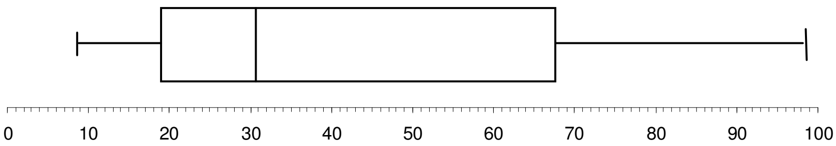
1.3



Note:
If just a box and whisker without any reference to the numbers: 1/3

✓ minimum and maximum values
✓ quartiles and median
✓ whiskers with median line
(3)

1.4

The data is skewed to the right (positively skewed).
This suggests that there was a large difference between the median and the maximum rainfall (some months had exceptionally high rainfall in that year).

✓ ✓ comment about rainfall.
(2)

Note:
Skewed to right 1/2

Die data is skeef na regs (positief skeef)
Dit dui daarop dat daar 'n groot verskil is tussen die mediaan en die maksimum reënval (sommige maande het ongewoon hoë reënval gehad gedurende die jaar.

✓ ✓ verwysing na reënval
(2)

1.5

By using the calculator, $\sigma = 28{,}19$. (28,19058256)

✓✓✓answer
Accept: 28 ; 28,2 ; 28,1
(3)

OR Pen and Paper method (not recommended)

Mean = 43,54 (43,54166667)

x	$x-\bar{x}$	$(x-\bar{x})^2$
60,9	17,36	301,3696
14,9	-28,64	820,2496
9,3	-34,24	1172,378
28,0	-15,54	241,4916
71,9	28,36	804,2896
76,4	32,86	1079,78
98,2	54,66	2987,716
65,7	22,16	491,0656
26,1	-17,44	304,1536
32,5	-11,04	121,8816
23,6	-19,94	397,6036
15,0	-28,54	814,5316
Sum		9536,509

✓headings correct
✓sum of the squares of the mean deviations

$$\sigma = \sqrt{\frac{9536{,}509}{12}} = 28{,}19$$ (28,19059…..)

✓ answer
(3)
[15]

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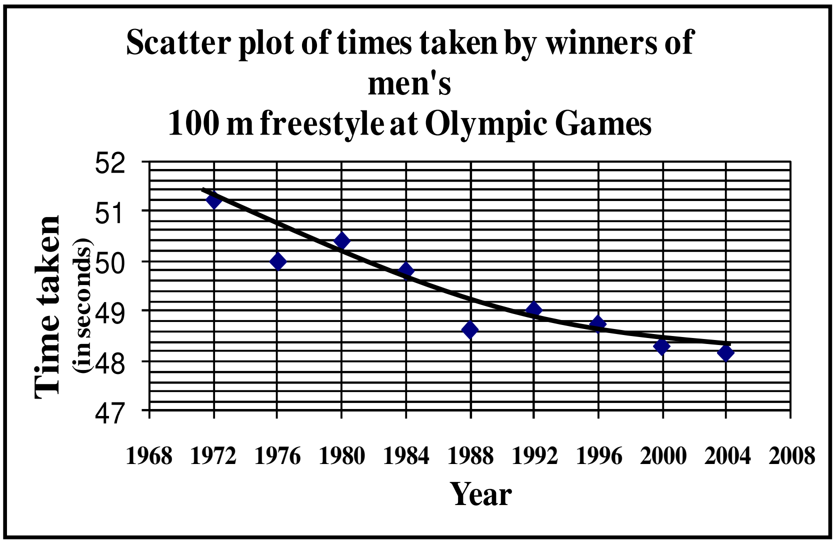
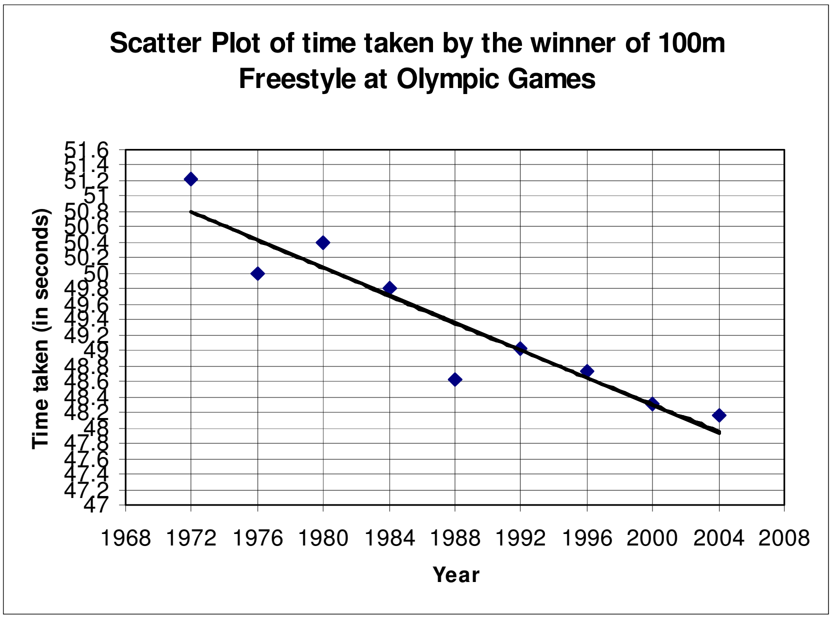
QUESTION 2

2.1	Linear or Exponential	✓ answer (1)
2.2	  For this set of data we will accept the straight line.	✓ ✓line of best fit (2)

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2.3	The scatter plot shows an overall decrease in the time taken by the winner since 1972. *Die spreidiagram dui 'n algehele afname in tye aangeteken deur die wenners vanaf 1972.* OR Times are faster. *Tye is vinniger.* OR Negative correlation between year and time. *Negatiewe korrelasie tussen jaar en tyd.*	✓ decrease/afname (1)
2.4	The top athletes of the world have turned professional. This allows them to train at the best facilities and receive the best coaching available. Also, equipment manufacturers are in competition with each other. In this case, manufacturers are designing swimsuits that assist swimmers Swimmers train harder and put in more effort. *Die top atlete van die wêreld het professionele atlete geword. Dit laat hulle toe om by die beste fasiliteite te oefen en die beste afrigting te ontvang.* *Vervaardigers van voorraad is in kompetisie met mekaar. Hul onwerp dus swembroeke wat die swemmers help.* *Swemmers oefen harder en gebruik meer tyd om te oefen.*	✓ any acceptable reason relating to the trend (1) ✓ *enige aanvaarbare rede wat verband hou met die neiging.* (1)
2.5	In the context of the times around these two observations, one can consider the efforts of 1976 and 1988 to be outliers. This shows that these athletes were exceptionally good swimmers at the time. *Binne die konteks van tye gedurende hierdie twee waarnemings, kan die poging van 1976 and 1988 gesien word as uitskieters. Dit dui daarop dat hierdie atlete uitstekende swemmers was daardie tyd.*	✓✓acceptable reason in context (2) ✓✓aanvaarbare rede binne die konteks (2)
2.6	Winning time of 2008 is expected to be about 47,6 seconds. Accept answer from candidate's graph.	✓answer from graph (1) **[8]**

QUESTION 3

3.1	50	✓ answer (1)
3.2	Cut–off mark of 56% (37 students)or 58% (38 students) Accept interval: 55% - 60%	✓ answer read off from ogive (1)

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3.3

Marks (out of 100)	Frequency (f)
0 ≤ marks <10	1
10 ≤ marks <20	3
20 ≤ marks <30	4
30 ≤ marks <40	11
40 ≤ marks <50	12
50 ≤ marks <60	9
60 ≤ marks <70	5
70 ≤ marks <80	4
80 ≤ marks <90	1
90 ≤ marks <100	0

✓ class intervals
Accept
0 – 10 ; 10 – 20
Or
0 < marks ≤ 10
Or
Between 0 and 10
Or
From 0 to 10

If the intervals not in tens, the mark for intervals not given

✓method
✓ accuracy of five answers
(3)
[5]

QUESTION 4

4.1	$\tan 45° = m_{AB}$ $= 1$ OR $m_{AB} = \frac{3-0}{1-t} = \frac{3}{1-t}$	✓ $\tan 45°$ ✓ answer (2) Answer only: full marks
4.2	$\frac{3-0}{1-t} = \tan 45° = 1$ $1-t = 3$ $t = -2$ OR $y = mx + c$ $3 = (1)(1) + c$ $c = 2$ $y = x + 2$ $(t;0)$ in $y = mx + 2$ $0 = t + 2$ $t = -2$	✓equating ✓value (2) ✓c=2 ✓value (2) Answer only: full marks

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4.3	$\sqrt{(1-p)^2+(3+4)^2}=\sqrt{50}$ $(1-p)^2+(3+4)^2=50$ $1-2p+p^2+49=50$ $p^2-2p=0$ $p(p-2)=0$ $p\neq 0$ or $p=2$ OR $(1-p)^2+(3+4)^2=50$ $(1-p)^2=50-49$ $(1-p)^2=1$ $1-p=1$ or $1-p=-1$ $p\neq 0$ $p=2$ OR *Let* $p=2$ $AC=\sqrt{(1-2)^2+(3+4)^2}$ $=\sqrt{1+49}$ $=\sqrt{50}$ which is true $\therefore p=2$	✓ substitution into distance formula ✓ expansion ✓factors ✓answer Note: If an answer was not chosen: 3/4 (4) ✓ substitution into distance formula ✓ expansion ✓factors ✓answer (4) If gradient of BC assumed as -1 and p calculated correctly: 0/4 Answer only: 1/4 ✓ substitution into distance formula ✓ $\sqrt{50}$ ✓which is true(justification) ✓answer (4) If equating to $\sqrt{50}$ from the start, then 3/4
4.4	midpoint of $BC=\left(\frac{-2+2}{2};\frac{0-4}{2}\right)$ midpoint of $BC=(0\,;-2)$	✓ x-value ($x=\frac{t+p}{2}$) ✓y-value (2)
4.5	Gradient of line = m_{AB} = 1 Equation of line is: $y + 4 = 1(x - 2)$ $y=x-6$ OR $y=mx+c$ $y=x-p-4$	✓ gradients are equal ✓substitution of (p;-4) ✓equation in any form (3) **[13]**

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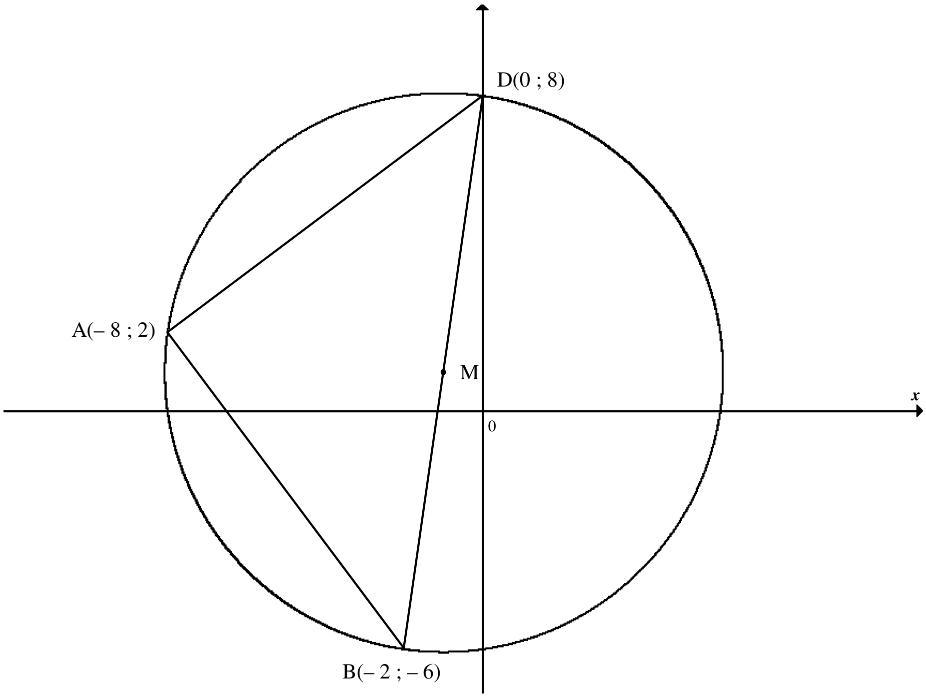
QUESTION 5



5.1	Midpoint BD $\left(\frac{0-2}{2};\frac{8-6}{2}\right)$ $=(-1;\ 1)$	✓x-coordinate ✓y-coordinate (2)
5.2	$y=7(-8)+58$ $=2$ $\therefore$ A lies on the line.	✓substitution (1) Substitute both at the same time with justification (1)
5.3	The line $y = 7x + 58$ is a tangent to the circle at A. $m_{line}=7$ $m_{AM}=\frac{2-1}{-8-(-1)}=-\frac{1}{7}$ $m_{line}\times m_{AM}=7\times-\frac{1}{7}=-1$ $\therefore$ AM $\perp$ to the line OR	✓relationship ✓✓ $m_{AM}=\frac{2-1}{-8-(-1)}=-\frac{1}{7}$ ✓ $m_{line}=7$ ✓product (5)

NOTE:
$m_{line}=7$ and CA gradient of AM then no relationship: 4/5

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5.3 contd	**OR** $m_{BD} = 7$ $m_{line} = 7$ $\therefore$ line // diameter **OR** $(x+1)^2 + (y-1)^2 = 50$ $x^2 + 2x + 1 + y^2 - 2y + 1 = 50$ $x^2 + 2x + 1 + (7x+58)^2 - 2(7x+58) + 1 = 50$ $x^2 + 2x + 1 + 49x^2 + 812x + 3364 - 14x - 116 + 1 = 50$ $50x^2 + 800x + 3200 = 0$ $x^2 + 16x + 64 = 0$ $(x+8)^2 = 0$ $x = -8$ $y = 2$ $y = 7x + 58$ is a tangent to the circle	✓✓ $m_{BD} = 7$ ✓ $m_{line} = 7$ ✓✓conclusion (5) Note: Only lines parallel 4/5 ✓ circle equation ✓substitution of $y = 7x + 58$ ✓standard form ✓ answer ✓ tangent (5)
5.4	$AD = \sqrt{(8-2)^2 + (0+8)^2}$ $= \sqrt{36+64}$ $= 10$ $AB = \sqrt{(2+6)^2 + (-8+2)^2}$ $= \sqrt{64+36}$ $= 10$	✓ substitution ✓ answer ✓ substitution ✓ answer (4) Note: Answers $\sqrt{10}$ then 3/4

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5.5	$m_{AD} = \frac{8-(2)}{0-(-8)}$ $m_{AD} = \frac{3}{4}$ $m_{AB} = \frac{2-(-6)}{-8-(-2)}$ $= -\frac{4}{3}$ $m_{AB}.m_{AD} = -\frac{4}{3} \times \frac{3}{4}$ $= -1$ $D\hat{A}B = 90°$	✓ gradient of AD ✓ gradient of AB ✓ PRODUCT (3)
	OR $BD^2 = (8+6)^2 + (0+2)^2$ $= 200$ $= AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$	✓ distance formula ✓ Pythagoras ✓ conclusion (3)
	OR $a^2 = b^2 + d^2 - 2(b)(d)\cos A$ $200 = 100 + 100 - 2(10)(10)\cos A$ $0 = -200\cos A$ $A = 90°$	✓ cos rule ✓ substitution ✓ conclusion (3)
	OR $(AD)^2 = 100$ $(AB)^2 = 100$ $BD^2 = (-2-0)^2 + (-6-8)^2$ $= 4 + 196$ $= 200$ $\therefore BD^2 = AD^2 + AB^2$ $\therefore D\hat{A}B = 90°$ (Pyth)	✓ $BD^2 = 200$ ✓ $BD^2 = AD^2 + AB^2$ ✓ conclusion (3)
	OR $\hat{A} = 90°$ (angles in semi - circle)	✓ ✓✓reason (3)
5.6	$\theta = 45°$	✓ answer (1)

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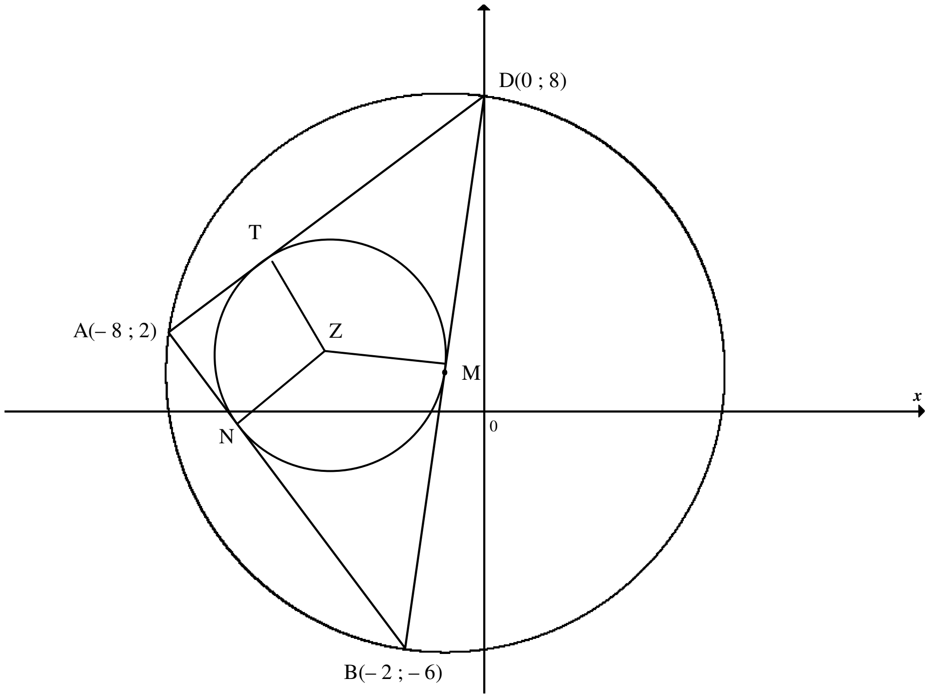

5.7	Let the radius of circle TNM be r NB = BM (properties of a kite) AN = TZ = r (TZNA is a square) NB = 10 – r BD = 2MB $\sqrt{(8-(-6))^2+(0-(-2))^2} = 2(10-r)$ $\sqrt{200} = 2(10-r)$ $10\sqrt{2} = 2(10-r)$ $r = 10-5\sqrt{2}$ $= 2{,}93$ **OR** $Z\hat{M}B = 90°$ $MB = \frac{1}{2}\sqrt{200}$ $= 7{,}07$ $\frac{ZM}{MB} = \tan 22{,}5°$ $ZM = 7{,}07 \tan 22{,}5°$ $= 2{,}93$ **OR**	✓ NB = BM ✓ AN = TZ = r ✓ NB = 10 – r ✓ BD = 2MB ✓BD = $\sqrt{200}$ ✓answer (6) ✓tan radius theorem ✓✓MB ✓✓tan 22,5° ✓answer (6)

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5.7 contd	$MB^2 = (-1+2)^2 + (1+6)^2$ $= 1 + 49$ $= 50$ $MB = \sqrt{50}$ $\frac{\text{ZM}}{\text{MB}} = \tan 22{,}5°$ $\text{ZM} = 7{,}07 \tan 22{,}5°$ $= 2{,}93$	✓✓MB ✓✓tan 22,5° ✓✓answer (6)
	OR By a well known formula Area ΔABD = r × (semi—perimeter) $\frac{1}{2} \times 10 \times 10 = r \times \frac{1}{2}(20 + \sqrt{200})$ $50 = r(10 + 5\sqrt{2})$ $r = 2{,}93$	✓✓ formula ✓ $\sqrt{200}$ ✓✓ answer (6)
	OR $MB = \sqrt{50}$ (radius of circle) $NB = \sqrt{50}$ (adjacent sides of kite) AB = 10 AN = $10 - \sqrt{50}$ = 2,93 But TANZ is a square ∴ AN = ZN ∴ radius = 2,93	✓MB ✓ NB ✓✓AN = 2,93 ✓ square ✓answer (6)

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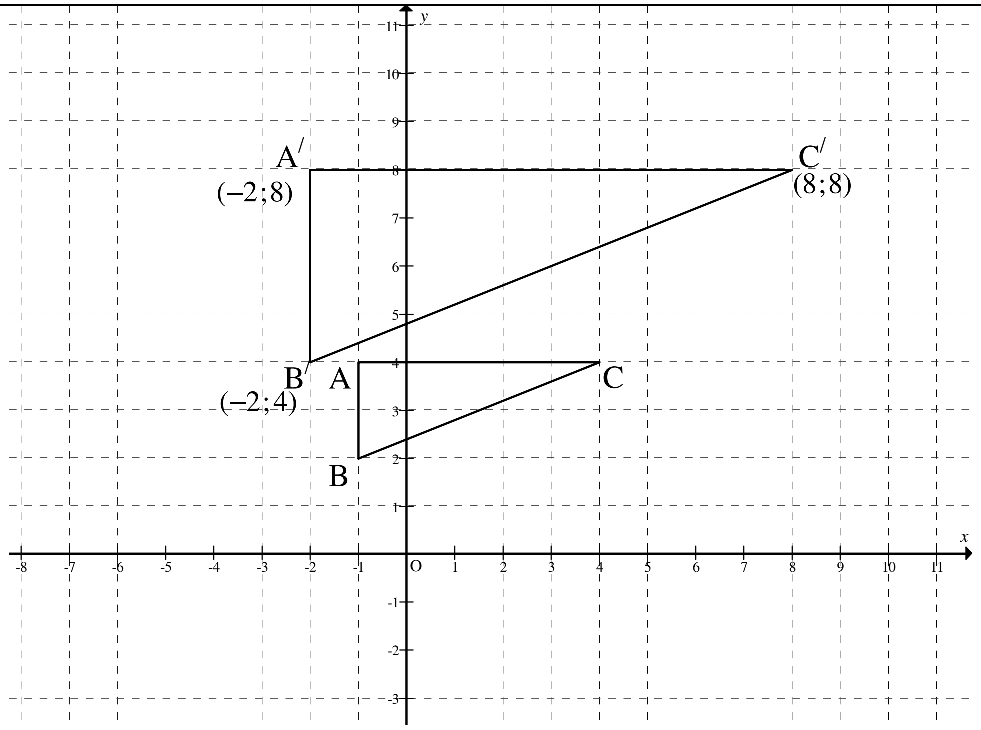
QUESTION 6

6.1.1	4×5 =20 squared units	✓✓answer $2^2 \times 5$ 1/2 If 2×5 = 10 0/2 (2)
6.1.2	$(x\,;\,y) \rightarrow (2x\,;\,2y)$ Note: If candidate state: coordinates times two 2/2	✓ $2x$ ✓ $2y$ (2) If $(kx\,;\,ky)$:1/2 If $2(x\,;\,y)$: 2/2
6.1.3		✓ coordinates $A^{/}$ ✓ coordinates $B^{/}$ ✓ coordinates $C^{/}$ (3) If diagram not drawn but coordinates correctly given: 1/3 If coordinates correctly plotted but not joined: 2/3
6.1.4	Not rigid. The shape remains the same, whilst the size is changed /enlarged **Note:** Shape remains the same: 1/2 Only the shape remains the same: 2/2	✓✓same shape and different size (2) not rigid only 2/2 just enlarged 0/2
6.2	Reflection about the line $y = x$: $(x\,;\,y) \rightarrow (y\,;\,x)$ Rotate clockwise about the origin: $(y\,;\,x) \rightarrow (x\,;-y)$ Translate 2 left and 3 down: $(x\,;-y) \rightarrow (x-2\,;-y-3)$ **OR** General rule: $(x\,;\,y) \rightarrow (x-2\,;-y-3)$	Mark per coordinate ✓✓ reflection ✓✓rotation ✓✓translation (6) **Answer only:** Full marks **[15]**

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	OR The first 2 transformations in the given order is the same as the reflection in the *x*-axis i.e. $(x\,;\,y) \rightarrow (x\,;\,-y)$ Then the translation gives us $(x\,;\,y) \rightarrow (x\,;\,-y) \rightarrow (x-2\,;\,-y-3)$ **NOTE:** If just given: $(x\,;\,y) \rightarrow (x-2\,;\,y-3)$: 2/6 If using $(x\,;\,y) \rightarrow (y\,;\,x)$ ✓✓ $(x\,;\,y) \rightarrow (y\,;\,-x)$ ✓ $(x\,;\,y) \rightarrow (x-2\,;\,y-3)$ ✓ throughout :4/6	If learner starts with $(x\,;\,y)$ and continue to use $(x\,;\,y)$ for the second and third transformation 4/6

QUESTION 7

7.1	$T^{/}\ (x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$	✓ x coordinate ✓ y coordinate (2) Clock-wise formula: 0/2
7.2	$A^{/}\ (p\cos135° - q\sin135°\,;\, q\cos135° + p\sin135°)$ If clockwise rotation: $A^{/}\ (p\cos135° + q\sin135°\,;\, q\cos135° - p\sin135°)$	✓ x coordinate ✓ y coordinate (2) CA from 7.1
7.3	$x' = p\cos(135°) - q\sin(135°)$ $-1-\sqrt{2} = -p\cos45° - q\sin45°$ $-1-\sqrt{2} = -p\left(\frac{\sqrt{2}}{2}\right) - q\left(\frac{\sqrt{2}}{2}\right)$ $-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}q$.................(1) and $y' = y\cos(135°) + p\sin(135°)$ $1-\sqrt{2} = -q\cos45° + p\sin45°$ $1-\sqrt{2} = q\left(-\frac{\sqrt{2}}{2}\right) + p\left(\frac{\sqrt{2}}{2}\right)$ $1-\sqrt{2} = -\frac{\sqrt{2}}{2}q + \frac{\sqrt{2}}{2}p$..................(2) (1) + (2): $-2\sqrt{2} = -\sqrt{2}q$ $q = 2$	✓ equating ✓ substitution ✓ equating ✓ substitution $\frac{\sqrt{2}}{2}$ ✓ solving simultaneously

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Substitute $q = 2$ into …..(1)

$-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}(2)$

$-1 = -\frac{\sqrt{2}}{2}p$

$p = \sqrt{2}$

$\therefore A = (\sqrt{2};2)$

Note: If not left in surd form: 6/7

OR

$x' = p\cos(135°) - q\sin(135°)$

$-1-\sqrt{2} = -p\cos 45° - q\sin 45°$

$-1-\sqrt{2} = -p\left(\frac{\sqrt{2}}{2}\right) - q\left(\frac{\sqrt{2}}{2}\right)$

$-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}q$.................(1)

and

$y' = y\cos(135°) + p\sin(135°)$

$1-\sqrt{2} = -q\cos 45° + p\sin 45°$

$1-\sqrt{2} = q\left(-\frac{\sqrt{2}}{2}\right) + p\left(\frac{\sqrt{2}}{2}\right)$

$-0,41 = -0,71q + 0,71p$..................(2)

(1) + (2):

$-2\sqrt{2} = -\sqrt{2}q$

$q = 2$

Substitute $q = 2$ into …..(1)

$-2,41 = -0,71p - 0,71q$ (2)

$1,42p = 2$

$p = 1,41$

$\therefore A = (\sqrt{2};2)$

Note: If not left in surd form: 6/7

OR

</td><td>

✓ answer for q

✓ answer for p

(7)

✓ equating

✓ substitution

✓ equating

✓ substitution $\frac{\sqrt{2}}{2}$

✓ solving simultaneously

✓ answer for q

✓ answer for p

(7)

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	$-\frac{\sqrt{2}}{2}(p+q)=-1-\sqrt{2}$ $p+q=-\frac{2}{\sqrt{2}}(-1-\sqrt{2})$ $p+q=\sqrt{2}+2$ and $\frac{1}{\sqrt{2}}(p-q)=1-\sqrt{2}$ $p-q=\sqrt{2}-2$ $p+q=\sqrt{2}+2$ $2p=2\sqrt{2}$ $p=\sqrt{2}$ $q=2$	✓ $-\frac{\sqrt{2}}{2}(p+q)=-1-\sqrt{2}$ ✓ substitution ✓ $\frac{1}{\sqrt{2}}(p-q)=1-\sqrt{2}$ ✓ substitution $\frac{\sqrt{2}}{2}$ ✓ solving simultaneously ✓ answer for q ✓ answer for p (7)
	OR A(p ; q) is obtained from A$'$ by a rotation through 135° in a clockwise direction $p=(-1-\sqrt{2})\cos(-135°)-(1-\sqrt{2})\sin(-135°)$ $=(-1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right)-(1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right)$ $=\frac{2}{\sqrt{2}}$ $=\sqrt{2}$ $q=(1-\sqrt{2})\cos(-135°)+(-1-\sqrt{2})\sin(-135°)$ $=(1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right)+(-1-\sqrt{2})\left(-\frac{1}{\sqrt{2}}\right)$ $=\frac{2\sqrt{2}}{\sqrt{2}}$ $=2$ $\therefore$ A $=(\sqrt{2};2)$	✓ substituting $(-1-\sqrt{2})$ ✓ substitution $\frac{1}{\sqrt{2}}$ ✓ equating ✓ substitution $\frac{1}{\sqrt{2}}$ ✓ substituting $(-1-\sqrt{2})$ ✓ answer for q ✓ answer for p (7)

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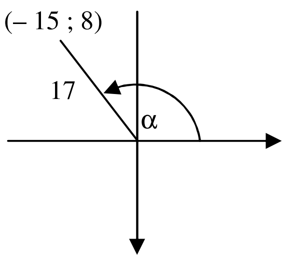
QUESTION 8

8.1	$\sin\alpha = \frac{8}{17}$  $\sin\alpha > 0 \therefore$ in second quadrant $y_\alpha = 8 \quad r_\alpha = 17$ $x_\alpha = -15 \quad (\text{Pythagoras})$ $\tan\alpha = -\frac{8}{15}$	$x = -\checkmark 15\checkmark$ ✓ answer (3) For drawing the radius vector in the correct quadrant 1/3 Without a sketch but correct values: 3/3
8.2	$\sin(90° + \alpha) = \cos\alpha$ $= -\frac{15}{17}$	✓ reduction ✓ answer (2) Answer only: full marks Cannot accept decimal values
8.3	$\cos 2\alpha = 1 - 2\sin^2\alpha$ $= 1 - 2\left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$ **OR** $\cos 2\alpha = 2\cos^2\alpha - 1$ $= 2\left(\frac{-15}{17}\right)^2 - 1$ $= \frac{161}{289}$ **OR** $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= \left(\frac{-15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$	✓ expansion ✓ substitution ✓ any further calculation or answer (3) ✓ expansion ✓ substitution ✓ any further calculation or answer (3) ✓ expansion ✓ substitution ✓ any further calculation or answer (3) **[8]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 9

NOTE: Only penalise once in the question for leaving out the x
Penalise once in this question for treating as an equation

9.1	$\sin(90° - x).\cos(180° - x) + \tan x.\cos(-x).\sin(180° + x)$ $= \cos x(-\cos x) + \tan x(\cos x)(-\sin x)$ $= -\cos^2 x - \frac{\sin x}{\cos x}\cos x \sin x$ $= -\cos^2 x - \sin^2 x$ $= -(\cos^2 x + \sin^2 x)$ $= -1$	✓ $\sin(90° - x) = \cos x$ ✓ $\cos(180° - x) = -\cos x$ ✓ $\cos(-x) = \cos x$ ✓ $\sin(180° + x) = -\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ simplification ✓ answer (7)
9.2	$\frac{\sin 190° \cos 225° \tan 390°}{\cos 100° \sin 135°}$ $= \frac{-\sin 10°(-\cos 45°)\tan 30°}{-\sin 10° \sin 45°}$ $= \frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$ or $= -\tan 30°$ $= -\frac{1}{\sqrt{3}}$ If using – cos 80° : no penalty If the candidate stop at $= \frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$ 6/7	✓ sin 190° = – sin 10° ✓ cos 225° = – cos 45° ✓ tan 390° = tan 30° ✓ cos 100° = – sin 10° ✓ sin 135° = sin 45° or cos 45° ✓✓ substitution (7)
9.3	$\sin x + 2\cos^2 x = 1$ $\sin x + 2(1 - \sin^2 x) = 1$ $-2\sin^2 x + \sin x + 1 = 0$ $2\sin^2 x - \sin x - 1 = 0$ $(2\sin x + 1)(\sin x - 1) = 0$ $\sin x = 1$ $x = 90° + k.360°; k \in Z$ Or	✓ substitution of identity ✓ standard form ✓ factorisation ✓ $\sin x = 1; \sin x = -\frac{1}{2}$ ✓ $x = 90° + k.360°; k \in Z$ ✓✓ answers (any two answers) (7) If $k \in Z$ not included: 6/7 Also ± $k.360°; k \in N_0$ or Z

- Consistent Accuracy will apply as a general rule.
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	$\sin x = -\frac{1}{2}$ $x = 210° + k.360°; k \in Z$ *OR* $x = 210° + k.360°$ or $x = 330° + k.360°; k \in Z$ *or* $x = -30° + k.360°$ OR $x = -150° + k.360°; k \in Z$ OR $x = -150° + k.360°; k \in Z$ *or* $x = 330° + k.360°$ *or* $x = -30° + k.360°$	✓ manipulation ✓ substitution of identity ✓ co ratios
	OR $\sin x + 2\cos^2 x = 1$ $\sin x = 1 - 2\cos^2 x$ $\sin x = -\cos 2x$ $\sin x = -[\sin(90° - 2x)]$ $x = 180° + (90° - 2x) + k360°$ $3x = 270° + k360°$ $x = 90° + k120°$ $k \in Z$ or $x = 360° - (90° - 2x) + k360°$ $x = -270° - k360°$	✓ $x = 180° + (90° - 2x) + k360°$ ✓ $x = 90° + k120°$ ✓ $x = 360° - (90° - 2x) + k360°$ ✓ $x = -270° - k360°$ (7) If $k \in Z$ not included: 6/7 ✓ manipulation ✓ substitution of identity ✓ co ratios
	OR $\sin x + 2\cos^2 x = 1$ $\sin x = 1 - 2\cos^2 x$ $\sin x = -\cos 2x$ $-\cos(90° - x) = \cos 2x$ $2x = 180° - (90° - x) + k360°$ $x = 90° + k360°$ $k \in Z$ or $2x = 180° + (90° - x) + k360°$ $3x = 270° + k360°$ $x = 30° + k120°$	✓ $2x = 180° - (90° - x) + k360°$ ✓ $x = 90° + k360°$ ✓ $2x = 180° + (90° - x) + k360°$ ✓ $x = 30° + k120°$ (7) If $k \in Z$ not included: 6/7 **[20]**

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 10

10.1	$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A.\cos B + \cos A.\sin B}{\cos A.\cos B - \sin A.\sin B}$ $= \frac{\sin A.\cos B + \cos A.\sin B}{\cos A.\cos B - \sin A.\sin B} \times \frac{\frac{1}{\cos A.\cos B}}{\frac{1}{\cos A.\cos B}}$ $= \frac{\frac{\sin A.\cos B}{\cos A.\cos B} + \frac{\cos A.\sin B}{\cos A.\cos B}}{\frac{\cos A.\cos B}{\cos A.\cos B} - \frac{\sin A.\sin B}{\cos A.\cos B}}$ $= \frac{\tan A + \tan B}{1 - \tan A.\tan B}$ **OR** $RHS = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$ $= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A}{\cos A}\frac{\sin B}{\cos B}} \times \frac{\cos A.\cos B}{\cos A.\cos B}$ $= \frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B}$ sin $= \frac{\sin(A+B)}{\cos(A+B)}$ $= \tan(A+B)$ = LHS	✓ expansions ✓ divisions ✓ tanA and tanB (3) ✓ $\frac{\sin A}{\cos A}$ ✓ multiplication ✓ expansions (3)
10.2	$\tan C = \tan(180° - (A+B))$ $\tan C = -\tan(A+B)$ $\tan C = -\left(\frac{\tan A + \tan B}{1 - \tan A.\tan B}\right)$ $\tan C(1 - \tan A.\tan B) = -(\tan A + \tan B)$ $\tan C - \tan A.\tan B.\tan C = -\tan A - \tan B$ $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$ **OR**	✓ C ✓ $-\tan(A+B)$ ✓ substitution into formula ✓ multiplication with LCD (4) If no conclusion: 3/4

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	$\hat{C} = 180° - (\hat{A} + \hat{B})$ (angles in a triangle) $\tan C = \tan(180° - (A + B))$ $\tan C = \tan((180° - A) + (-B))$ $\tan C = \frac{\tan(180° - A) + \tan(-B)}{1 - \tan(180° - A).\tan(-B)}$ $\tan C(1 - \tan(180° - A).\tan(-B)) = \tan(180° - A) + \tan(-B)$ $\tan C - \tan C \tan A \tan B = -\tan A - \tan B$ $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$	✓ C ✓ rearrange angle ✓ substitution into formula ✓ expansion (4)

QUESTION 11

NOTE: Penalty of one for early rounding off once in this question

11.1.1	$\text{B}\hat{\text{D}}\text{A} = 208° - 67°$ $= 141°$ $\frac{\sin \text{D}\hat{\text{B}}\text{A}}{97} = \frac{\sin 141°}{120}$ $\sin \text{D}\hat{\text{B}}\text{A} = 0{,}5087006494...$ $\text{D}\hat{\text{B}}\text{A} = 30{,}58°$ $\therefore$Bearing of Ship A from Ship B $= 180° - (360° - 208°) + 30{,}58°$ $= 58{,}58°$ **OR** $\text{B}\hat{\text{D}}\text{A} = 208° - 67°$ $= 141°$ $\frac{\sin D\hat{B}A}{97} = \frac{\sin 141°}{120}$ $\sin D\hat{B}A = 0{,}5087006494....$ $D\hat{B}A = 30{,}58°$ *then* $360° - 208° = N\hat{D}B$ (reflex angles) $\therefore N\hat{D}B = 152°$ *but* $M\hat{B}D + N\hat{D}B = 180°$ (co - interior angles/ angles around a point) $\therefore M\hat{B}D = 28°$ *then* $M\hat{B}A = M\hat{B}D + D\hat{B}A$ $= 30{,}58° + 28°$ $= 58{,}58°$	✓ $\text{B}\hat{\text{D}}\text{C} = 141°$ ✓ sine rule ✓ substitution ✓ $\hat{\text{B}} = 30{,}58°$ ✓ method or $\text{M}\hat{\text{B}}\text{D} = 28°$ ✓ answer (6) ✓ $\text{B}\hat{\text{D}}\text{C} = 141°$ ✓ sine rule ✓ substitution ✓ $N\hat{D}B = 152°$ ✓ $M\hat{B}D = 28°$ ✓ answer (6)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
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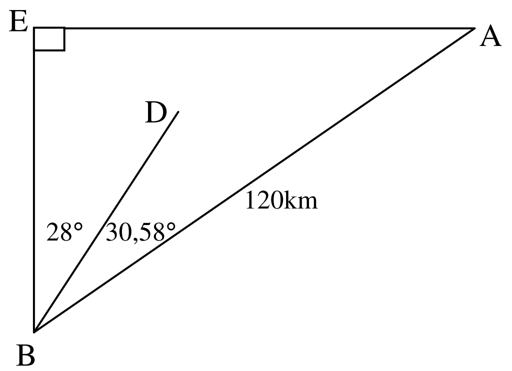
11.1.2

$\hat{B} = 30{,}58°$



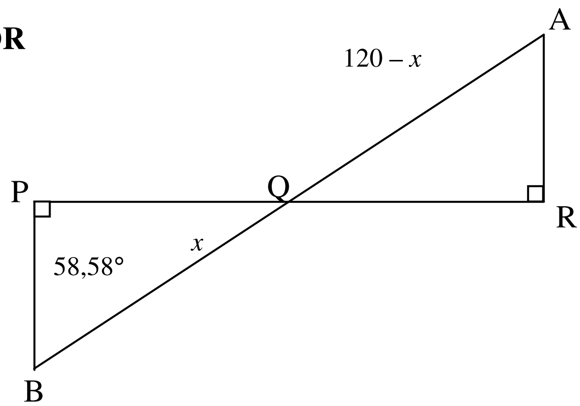
$\frac{EA}{120} = \sin(28° + 30{,}58°)$

$EA = 120\sin(28° + 30{,}58°)$ ✓ definition ✓ substitution

$EA = 102{,}4$ km ✓ answer

(3)

OR



Let BQ = x, then AQ = $120 - x$

$\sin 58{,}58° = \frac{PQ}{x}$ $\sin 58{,}58° = \frac{QR}{120 - x}$

$PQ = x.\sin 58{,}58°$ $QR = (120 - x)\sin 58{,}58°$ ✓ trigonometeric ratios

$PQ + QR = x.\sin 58{,}58° + (120 - x)\sin 58{,}58°$

$= 120\sin 58{,}58°$ ✓ sum

$= 102{,}4$ ✓ answer

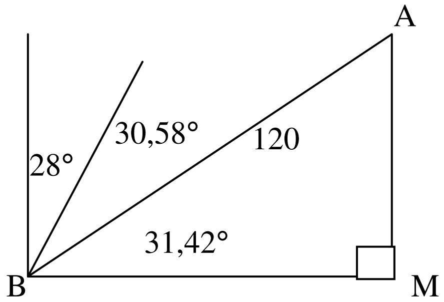
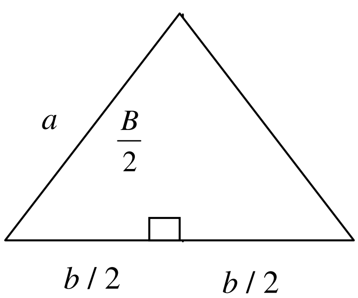
(3)

OR

BP = AR (assume ships move at same speed)

✓ trigonometeric

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

	$\Delta PBQ \equiv \Delta RAQ$ (angle, angle, side) $\therefore$ BQ = QA = 60 km $\sin 58{,}58° = \frac{PQ}{60}$ $\therefore PQ = 60\sin 58{,}58°$ $= 51{,}20$ km PR = 2PQ $= 102{,}4$ km OR $\frac{BM}{120} = \cos 31{,}42$ $BM = 120 \times \cos 31{,}42°$ $= 102{,}4$  	ratios ✓ 51,20 km ✓ answer (3) ✓ trigonometeric ratios ✓ substitution ✓ answer (3)
11.2	AB = BC = $a = c$ $b^2 = a^2 + c^2 - 2ac \times \cos B$ $b^2 = a^2 + a^2 - 2a \times a \times \cos B$ $b^2 = 2a^2 - 2a^2 \cos B$ $b^2 = 2a^2(1 - \cos B)$ $\frac{b^2}{2a^2} = 1 - \cos B$ $\cos B = 1 - \frac{b^2}{2a^2}$ **OR** $\sin\frac{B}{2} = \frac{b}{2a}$ $\cos B = 1 - 2\sin^2\frac{B}{2}$ $= 1 - 2\left(\frac{b}{2a}\right)^2$ $= 1 - \frac{b^2}{2a^2}$  	✓ equal sides ✓ cos rule ✓ substitution ✓ simplification (4) ✓ $\sin\frac{B}{2}$ ✓ $\sin\frac{B}{2} = \frac{b}{2a}$ ✓ formula ✓ substitution (4) **[13]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
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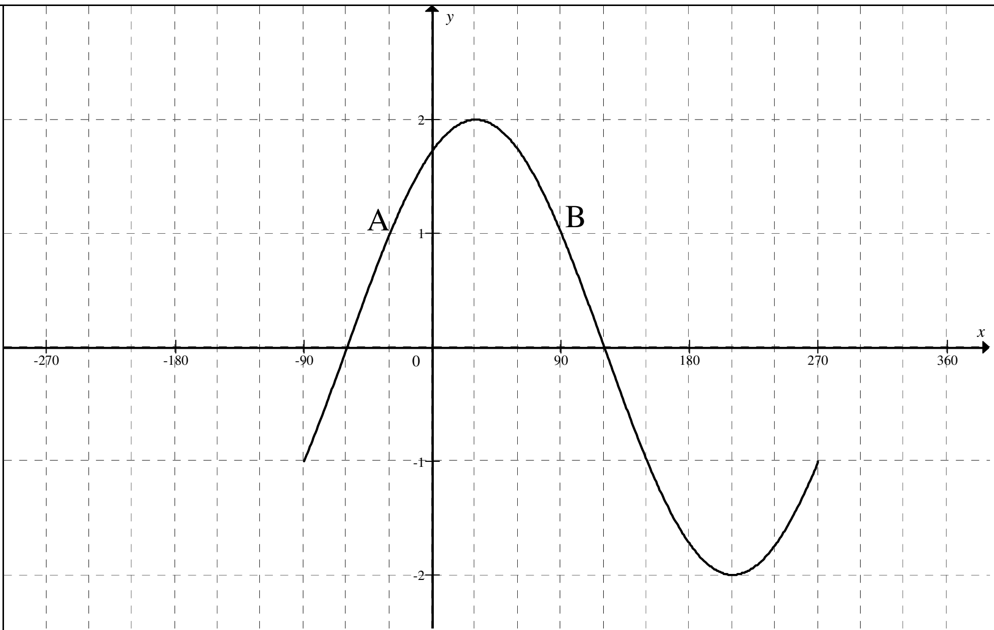
	OR $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ $but\ a = c$ $\cos B = \frac{a^2 + a^2 - b^2}{2a.a}$ $= \frac{2a^2 - b^2}{2a^2}$ $= 1 - \frac{b^2}{2a^2}$	✓ cos rule ✓ equal sides ✓ substitution ✓ simplification (4)

QUESTION 12

12.1		✓ (120°; 0) or (–60°; 0) ✓ (30°; 2) or (210°; – 2) (2)

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 12

12.2	$\cos(x-30°)=\frac{1}{2}$ $2\cos(x-30°)=1$ See points A and B on the graph **Note:** If drawn the line y = $\frac{1}{2}$ and put A and B on the graph: 0/2 If A and B on the *x*-axis: 1/2 If $A=-30°$ and $B=90°$: 1/2	✓manipulation ✓ answer (2) A and B in the correct place on the graph: full marks
12.3	$\cos(x-30°)=0,5$ $x-30°=60°$ OR $x-30°=-60°$ $x=90°$ $x=-30°$	✓ 60° (ref angle) ✓ 90° ✓ – 30° (3) Answer only: 3/3
12.4	$g'(x)=0$ is at maximum and minimum values of graph x = 30°; 210°	✓✓ one for each *x*-value (2)
12.5	$x\in[-90°\,;-60°)\cup(120°\,;270°]$ **OR** $-90°\le x<-60°$ or $120°<x\le 270°$ **OR** *If* $x<-60°$ *or* $x>120°$ 2/3	✓ notation ✓✓ critical values (3) **[12]**