NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2009

MATHEMATICS: PAPER I

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 pages, an Answer Booklet of 4 pages (i – iv) and an Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions. Questions 6 and 7 should be answered in the Answer Booklet.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a) Solve for x:

(1) $8x^2 + 1 = 7x$ (4)

(2) $x^2 \geq 81$ (4)

(3) $\log 10^{x-5} = 7$ (2)

(b) $(4\,;0)$ is a point on the curve with equation $y = x^2 - kx - 12$. Determine the value of k. (3)

(c) The first three terms of a geometric sequence are $6;\ x;\ 54$.
Determine the value(s) of x. (3)

16 marks

QUESTION 2

(a) Given $f(x) = 2x$, determine $f'(x)$ **from first principles**. (4)

(b) Writing your answer with positive exponents, find $\dfrac{dy}{dx}$ if:

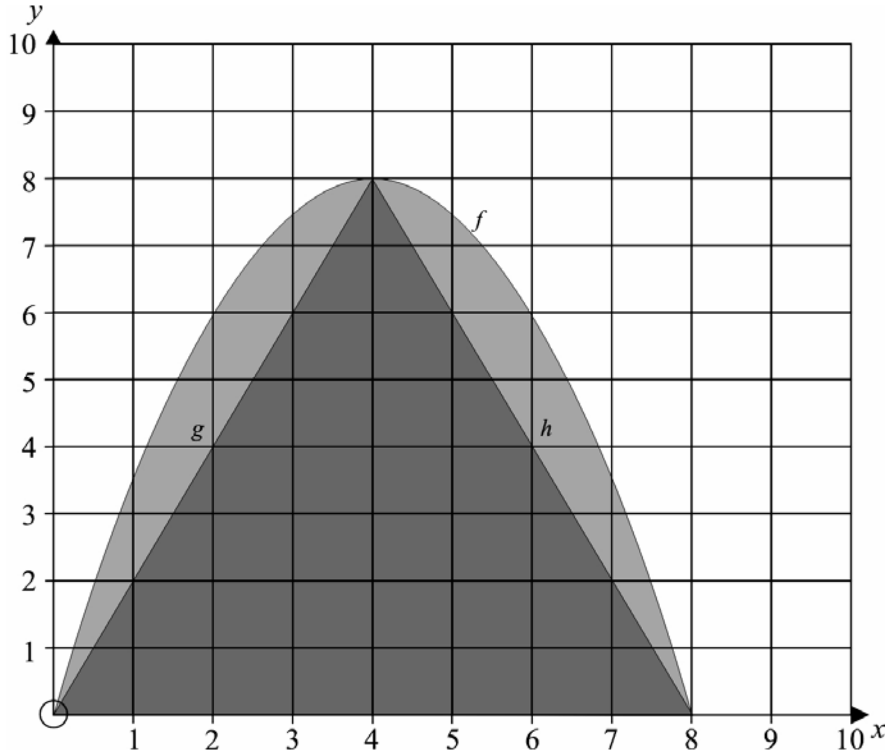
(1) $y = \dfrac{x^3}{6} - \dfrac{6}{x^2}$ (3)

(2) $y = \dfrac{x^2 - 8x + 12}{3x - 6}$ (3)

10 marks

QUESTION 3

(a) A ballistics manufacturer is designing the head of a new missile. The following diagram illustrates the cross-section of the head with triangular central shaft coated with another material forming a parabola.



The equations of the parabola f and two straight lines g and h need to be stored in the machinery. Determine these equations and write down the domain of h. (7)

(b) The graph with equation $y = x^2$ is translated 3 units down parallel to the y-axis and 2 units to the right parallel to the x-axis. Write down the equation of the new graph. It is not necessary to simplify it. (2)

(c) Given: $f(x) = \dfrac{8x + 32}{20}$ and $g(x) = \dfrac{5x}{2} - 4$

(1) Calculate: $f(1) + g(2)$. (2)

(2) Verify that g is the inverse of f. (3)

(d) Examine the following table.

A	A hyperbola passing through (4 ; 4).
B	A circle (centre the origin) with radius 4 units.
C	A parabola turning at the origin and passing through (4 ; 4).
D	A semi-circle (centre the origin) below the *x*-axis with radius 4 units.

Choose one equation from the following table that matches each description above.

P	$x^2 + y^2 = 4$
Q	$4y = x^2$
R	$xy = 16$
S	$\frac{x^2}{2} + \frac{y^2}{2} = 8$
T	$y = \frac{4}{x}$
U	$y = 4x^2$
V	$y = -\sqrt{16 - x^2}$

Your answer should show a letter A to D linked to a letter from the second list P to V. One of the possible answers, which need not be the correct one, for example, is A – Q. (4)

18 marks

QUESTION 4

(a) Jean borrows R450 000 to buy a home.
The interest rate is 15,5% per annum, compounded monthly on the outstanding balance over the life of the loan.
She will fully repay the loan over 20 years with equal monthly instalments, starting one month after securing the loan.

Determine:

(1) her monthly payments. (5)

(2) the total amount of interest she will pay on the loan. (2)

(b) Nancy inherits R250 000 and decides to invest it in a savings account earning interest at 5,8% p.a. compounded monthly.

(1) Calculate how many years it will take for her investment to be worth R500 000. Give your answer to the nearest year. (4)

(2) Determine the effective annual interest rate (correct to one decimal digit) for Nancy's investment. (2)

(3) The 'Rule of 72' is a rough guide used to estimate the number of years needed for an investment (or population) to double in size. The rule is:

$$\text{Number of years} = \frac{72}{r}$$

where r is the effective annual interest rate.

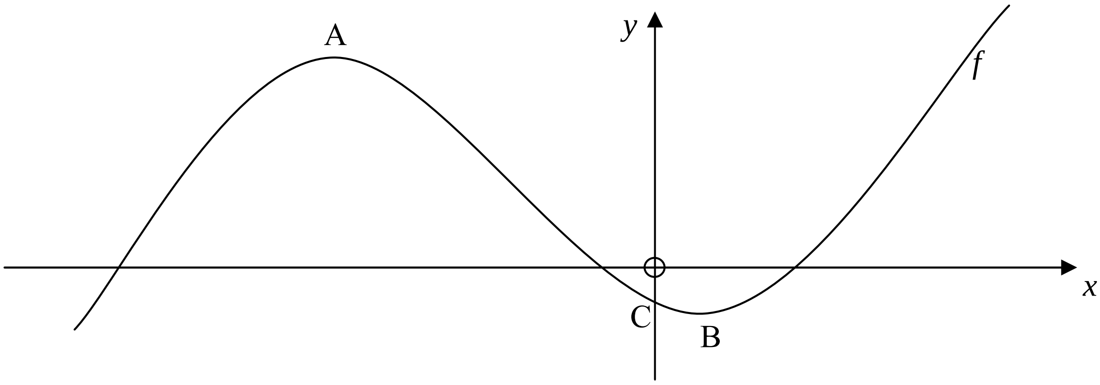
Determine how this estimate compares with your answer in (1) for Nancy's investment. (2)

(c) A shop owner adds 25% to the cost price of items and then gives customers a discount on the marked price. After this, he still makes a profit of 5% of the original cost price. Determine this discount as a percentage of the marked price. (5)

20 marks

QUESTION 5

Refer to the figure.
The graph (not drawn to scale) of $f\ (x) = 4x^3 + 27x^2 - 30x - 1$ is shown with A and B the turning points of the graph.



(a) Determine the coordinates of A and B. (6)

(b) Calculate the average gradient of f between the points A and B. (2)

(c) C is the y-intercept of the graph. Determine the equation of the tangent to f at C. (3)

(d) Determine the x-coordinate of the point on f where this tangent cuts the graph again. (3)

14 marks

SECTION B

QUESTION 6 ANSWER IN THE ANSWER BOOKLET PROVIDED

(a) The following constraints apply to a linear programming problem.

$$x \geq 0, \quad y \geq 0, \quad x + y \geq 50, \quad x + y \leq 100, \quad y \leq x$$

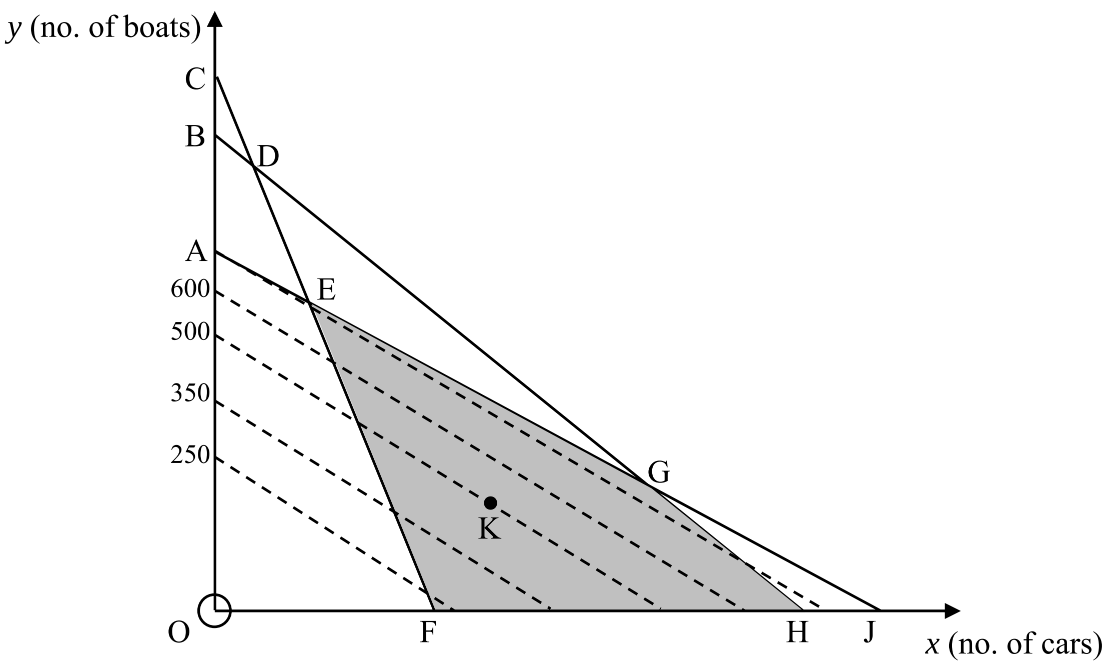
(1) Draw these constraints on the axes provided and shade the feasible region. (5)

(2) Which constraint does not influence the feasible region? (1)

(3) Give the maximum value of y that satisfies all the constraints. (1)

(4) Given the objective function $C = 2x + y$ determine the minimum value of C that satisfies all the constraints. (3)

(b) A linear programming technique has been applied to a situation where the variables are the number of cars (x) and the number of boats (y) produced by a toy manufacturer. The shaded area below is the feasible region:



Assume that all points A to K have integer coordinates.

Suppose that c_1 is the profit made on each car and c_2 is the profit made on each boat. Hence the values c_1 and c_2 can be assumed to be positive constants.

The profit function is $P = c_1 x + c_2 y$
and is represented on the graph above by dotted lines.

(1) Write down the profit obtained at the point K in terms of c_2. (1)

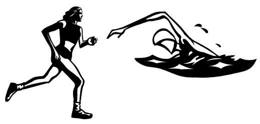
(2) Identify the point in the feasible region that yields the maximum profit. (1)

(3) As a result of an improved manufacturing process, the profit made on each car increases to become c_3, whereas the profit on each boat remains constant, such that the profit function is now $P = c_3 x + c_2 y$.

Determine which labelled point(s) in the feasible region are now likely to maximise the profit. (3)

15 marks

QUESTION 7 ANSWER IN THE ANSWER BOOKLET PROVIDED



Alex decides to include both swimming and running in her exercise plan.
On day 1, Alex swims 100 m and runs 500 m.
Each day she will increase the distance she swims by 50 m and the distance she runs will increase by 3,5% of the distance she ran on the previous day.

(a) Determine, in terms of n, the distance that Alex

(1) swims on the n^{th} day of her exercise plan.
(2) runs on the n^{th} day of her exercise plan. (6)

(b) On the set of axes provided, plot points for each of the exercise types. You may join the points to illustrate the trend. (5)

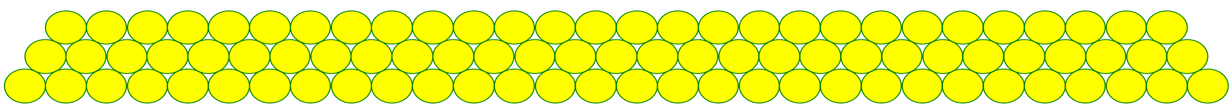
(c) **Use your graphs** to determine the first day on which the distance Alex swims will be greater than the distance she runs. (1)

12 marks

QUESTION 8

(a) Evaluate: $\sum_{k=1}^{\infty} \frac{2}{3^k}$ (3)

(b) Refer to the figure showing the first three layers of a stack of cans.
There are 30 cans in the first layer, 29 cans in the second layer and 28 cans in the third layer.



This pattern of stacking cans in layers continues.

(1) Determine a formula for the number of cans in the n^{th} layer. (2)

(2) Determine the maximum number of cans that can be stacked in this way. (4)

9 marks

QUESTION 9

(a) Given: $f(x) = \sqrt{4x}$ and $g(x) = x^2$
Evaluate $f(g(9))$. (3)

(b) Sam claims that the graph of $h(x) = \dfrac{x-1}{x-2}$
is a hyperbola with horizontal asymptote $y = 1$.

(1) Give an algebraic manipulation of h that Sam might use to prove his claim. (3)

(2) Draw a sketch of $y = h(x)$, clearly showing intercepts with the axes and asymptotes. (5)

(3) Use your sketch to determine the values of x for which $\dfrac{x-1}{x-2} \leq 0$. (1)

12 marks

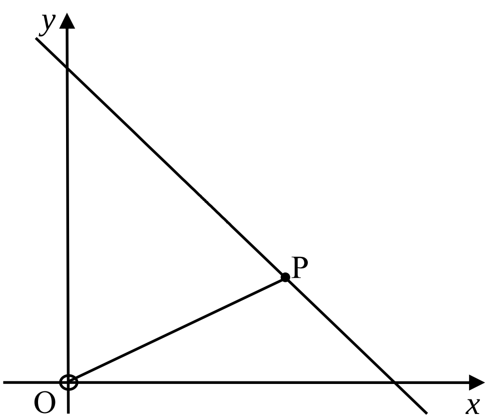
QUESTION 10

(a) A company determines that the profit (in Rands) of producing x units of a certain product is :

$$\text{P}(x) = 50\sqrt{x} - 0{,}5x - 500$$

Calculate the number of units that need to be produced so that the maximum profit is generated. (5)

(b) Refer to the figure.
The point P lies on the straight line $2x + y = 10$.



(1) Show that
$\text{OP}^2 = 5x^2 - 40x + 100$. (3)

(2) Determine the coordinates of P that minimise OP^2. (4)

(3) Hence, or otherwise, calculate the minimum length of OP. (2)

14 marks

QUESTION 11

Consider the following list of numbers:

41, 43, 47, 53, 61, 71, 83, 97, 113, 131,
151, 173, 197, 223, 251, 281, 313, 347, 383, 421

(a) This sequence has a constant second difference. Determine a formula to represent the n^{th} term. (4)

(b) An interesting property of this sequence is that the first 40 terms are all prime numbers. Using your formula, calculate the 41^{st} term of the sequence and show that it is not a prime number. (3)

(c) Determine the units digit of the $49\ 999\ 998^{th}$ term of the sequence. (3)

10 marks

Total: 150 marks