NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2008

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1)

$x^2 = 5x + 6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$ ✓A

$x = 6$ or $x = -1$ ✓CA (2)

(2) $3^x = 33$

$x = \log_3 33$ ✓M Introducing logs

$= 3{,}2$ ✓A (2)

(3) $x^2 - x < 12$

$x^2 - x - 12 < 0$

$(x-4)(x+3) < 0$ ✓A

Sign diagram on a number line with -3 and 4 marked: $+$ 0 $-$ 0 $+$ ✓M ✓A Signs

$-3 < x < 4$ ✓CA (4)

(b) $2x = 3x^2 - 1$

$3x^2 - 2x - 1 = 0$ ✓M Equating

$(3x+1)(x-1) = 0$ ✓A

$x = -\frac{1}{3}$ or $x = 1$ ✓A

$y = -\frac{2}{3}$ or $y = 2$ ✓CA (4)

(c) (1) $f(x) = x^3 + x^2 - 3x - 3$

$= x^2(x+1) - 3(x+1)$ ✓M ✓A Grouping

$= (x+1)(x^2-3)$ ✓A (3)

ALTERNATIVELY :

$f(-1) = -1+1+3-3$ ✓M Factor Theorem

$= 0$

$\therefore\ x+1$ is a factor

$f(x) = (x+1)(x^2-3)$ ✓A ✓A

(2) (i) $x = -1$ ✓A (1)

(ii) $f(x) = (x+1)(x-\sqrt{3})(x+\sqrt{3})$

$\therefore x = -1,\ x = +\sqrt{3},\ x = -\sqrt{3}$ ✓A✓A (2)

18 marks

QUESTION 2

(a) A.P. $a = d = 2$ ✓A

$T_n = 2n + 1$ ✓M

$A = T_7$ ✓M Sub into T_7

$= 2 \times 7 + 1$

$= 15$ ✓A

$2B + 1 = 153$ ✓M Setting = 153

$2B = 152$

$B = 76$ ✓A (6)

(b) A.P. $a = 7$; $T_{15} = 35$

$7 + 14d = 35$ ✓M Formula for T_{15}

$14d = 28$

$d = 2$ ✓A

$T_8 = a + 7d$

$= 7 + 7 \times 2$ ✓M Sub in T_8

$= 21$ ✓A (4)

or $\dfrac{T_{m+n}}{2} = \dfrac{T_m + T_n}{2}$ ✓M

$\therefore T_8 = \dfrac{T_1 + T_{15}}{2}$ ✓M

$= \dfrac{7 + 35}{2}$ ✓A

$= 21$ ✓A

(c) (1) $\sum_{x=1}^{4}\left(x^2 - x + 1\right)$

$= 1 + 3 + 7 + 13$ ✓M ✓A Expanding

$= 24$ ✓CA (3)

(2) $\dfrac{45}{4} + \dfrac{135}{16} + \dfrac{405}{64} + \dfrac{1215}{256} + \cdots$ to 15 terms

G.S. $a = \dfrac{45}{4}$; $r = \dfrac{3}{4}$ ✓A

$S_{15} = \dfrac{a\left(1 - r^{15}\right)}{1 - r}$ ✓M Sub into S_{15}

$= \dfrac{\dfrac{45}{4}\left\{1 - \left(\dfrac{3}{4}\right)^{15}\right\}}{1 - \dfrac{3}{4}}$

$= 44,3986 \ldots$ ✓A

$\approx 44,4$ ✓CA (4)

17 marks

QUESTION 3

(a) (1)

$F_v = 255\,000(1-0{,}125)^3$ ✓M — Sub into depreciation formula

$= \text{R } 170\,830{,}08$ ✓A

(2)

(2)

$100\,000 = 255\,000(1-0{,}125)^n$ ✓M — Sub into depreciation formula

$0{,}3921568627 = 0{,}875^n$ ✓A

$n = \log_{0{,}875}(0{,}392...)$ ✓M — Introducing logs

$= 7{,}010286801$ ✓A

≈ 7 years ✓CA

(5)

(b)

$\frac{1}{2} = (1-i)^6$ ✓A

$1-i = \sqrt[6]{0{,}5}$ ✓M — Simplifying

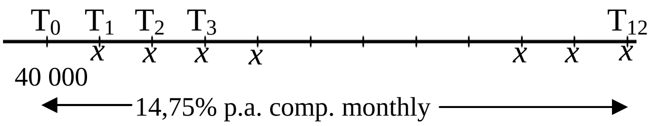
$= 0{,}8908987181...$ ✓A

$-i = -0{,}1091...$

$i \approx 10{,}9\%$ ✓CA

(4)

(c)



$\left(1+\frac{0{,}1475}{12}\right)^{-1} = 0{,}987857\ldots$ ✓A ✓M

$40\,000 = x(0{,}98\ldots) + x(0{,}98\ldots)^2 + \ldots + x(0{,}98\ldots)^{12}$ ✓A ✓M — Series

$= \frac{x(0{,}98...)\{1-(0{,}98...)^{12}\}}{1-0{,}98...}$ ✓A

$= x\{11{,}0938...\}$ ✓A

$x = 3605{,}615478\ldots$

$\approx \text{R}3\,605{,}62$ ✓CA

(7)

ALTERNATIVELY:

$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ ✓A

$40\,000 = \frac{x\left[1-\left(1+\frac{0{,}1475}{12}\right)^{-12}\right]}{\frac{0{,}1475}{12}}$ ✓M ✓A — Formula

$= x\{11{,}0938...\}$ ✓A ✓M

$x = 3605{,}615478...$ ✓A

$\approx R3605{,}62$ ✓CA

18 marks

QUESTION 4

(a) $y = 3x^3 + x$

$\frac{dy}{dx} = 9x^2 + 1$ ✓A ✓A (2)

(b) $f(x) = \frac{1}{2\sqrt{x}}$

$= \frac{1}{2}x^{-\frac{1}{2}}$ ✓A

$f'(x) = \frac{1}{2}\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$ ✓M Differentiating

$= -\frac{1}{4x^{\frac{3}{2}}}$ ✓A

$f'(4) = -\frac{1}{4\times 4^{\frac{3}{2}}}$

$= -\frac{1}{32}$ ✓A (4)

(c) $y = x^3 - x^2 - x + 2$

(1) $y = (-1)^3 - (-1)^2 - (-1) + 2$ ✓M Sub -1 in original

$= -1 - 1 + 1 + 2$

$= 1$ ✓A

$\frac{dy}{dx} = 3x^2 - 2x - 1$ ✓A

$m = 3(-1)^2 - 2(-1) - 1$ ✓M Sub -1 into derivative

$= 3 + 2 - 1$

$= 4$ ✓A

Eqn. of tangent: $y - 1 = 4(x+1)$ ✓M Str. line formula

$y = 4x + 4 + 1$

$y = 4x + 5$ ✓A (7)

(2) $3x^2 - 2x - 1 = 4$ ✓M Derivative = grad. from (1)

$3x^2 - 2x - 5 = 0$

$(3x-5)(x+1) = 0$ ✓A

$x = \frac{5}{3}$ or $x = -1$ ✓A (3)

16 marks

QUESTION 5 In Answer Booklet – Marking Guidelines **[11]**

QUESTION 6 In Answer Booklet – Marking Guidelines **[16]**

QUESTION 7

(a) $a^x = 1$

(1) $a^0 = 1$
$a \, \varepsilon \, R, \quad a \neq 0$ ✓A (1)

(2) $a^x = 1$
$a = 1$ ✓A (1)

(b) $x = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$

(1) $x^2 = 6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$ ✓M Squaring
$x^2 = 6 + x$ ✓A (2)

(2) $x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$ ✓A
$x = 3$ or $x \neq -2$
✓A N.V.✓M (3)

(c) $f(x) = 3x$
$f^{-1}: \; x = 3y$ ✓M $x \Leftrightarrow y$
$y = \dfrac{x}{3}$ ✓A

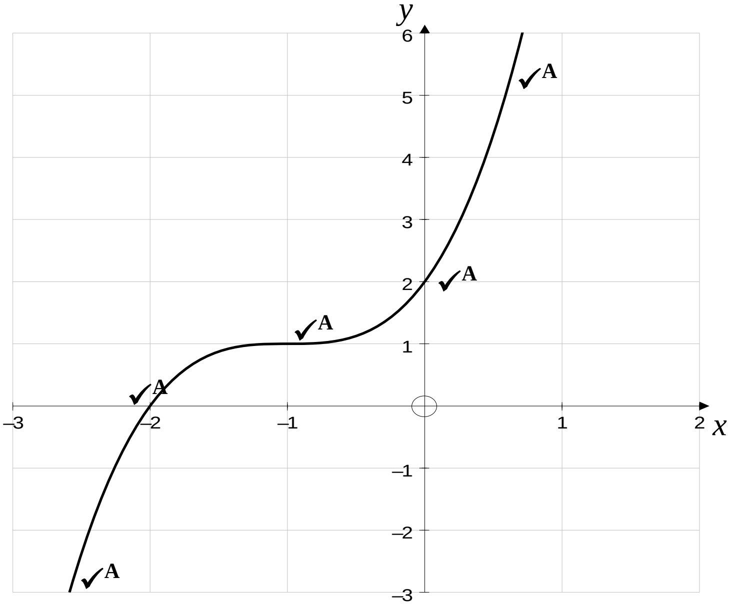
$f(x) + f\left(\dfrac{1}{x}\right) + \dfrac{1}{f(x)} + f^{-1}(x)$ ✓A

$= 3x + \dfrac{3}{x} + \dfrac{1}{3x} + \dfrac{x}{3}$ ✓A

$= \dfrac{9x^2 + 9 + 1 + x^2}{3x}$ ✓M Taking LCD

$= \dfrac{10(x^2+1)}{3x}$ ✓A (6)

(d)



(5)

18 marks

QUESTION 8

(a) (1) $y = \dfrac{-x^2}{240} + 210$

Highest point on arch is when $x = 0$ ✓M

i.e. $y = 210$ ✓A

$\therefore$ Bridge is $210 + 6 = 216$ m above gorge. (2)

(2) Span is 272 m.

Arch reaches cliff at $x = 136$ m.

$y = -\dfrac{136^2}{240} + 210$ ✓M Sub. in 136

$= 132{,}9\dot{3}$ ✓A

Longest Pillar $= 216 - 132{,}9\dot{3}$ ✓M Difference

$= 83{,}0\dot{6}$

$\approx 83{,}1$ m ✓A (4)

(b) Distance $= 160 + 2\left(\dfrac{2}{3}\times 160 + \left(\dfrac{2}{3}\right)^2 \times 160 + \ldots\right)$

$S_\infty = \dfrac{\frac{2}{3}\times 160}{1-\frac{2}{3}}$ ✓M ✓A Sum to infinity

$= 320$ ✓A

$\therefore$ Total Vert. Dist. $= 160 + 2 \times 320$ ✓A

$= 800$ m ✓CA (5)

11 marks

QUESTION 9

(a) (1) $y = 2x^3 - 17x^2 + 35x = 0$ ✓M — $y = 0$

$x(2x^2 - 17x + 35) = 0$

$x(2x-7)(x-5) = 0$ ✓A

$x = 0$ or $x = \frac{7}{2}$ or $x = 5$ ✓A

Tunnel is $\frac{7}{2} \times 100$ ✓M

$= 350$ m ✓A (5)

(2) Bridge is $\left(5 - \frac{7}{2}\right) \times 100$

$= 150$ m ✓A (1)

(b) $\frac{dy}{dx} = 6x^2 - 34x + 35 = 0$ ✓A ✓M — Derivative = 0

$x = \frac{34 \pm \sqrt{34^2 - 24 \times 35}}{12}$ ✓M — Quadratic for

$= 4{,}31469907$ or $1{,}351967597$ ✓A

$y = 2(1{,}35...)^3 - 17(1{,}35...)^2 + 35(1{,}35...)$ ✓M — Sub. 1,35 …

$= 21{,}18828443$ ✓A

$\therefore$ Mountain is 2119 m high ✓CA (7)

(c) $y = 2(4{,}31...)^3 - 17(4{,}31...)^2 + 35(4{,}31...)$ ✓M — Sub. 4,31 …

$= -4{,}817914055$ ✓A

$\therefore$ Drop is 482 m ✓CA (3)

16 marks

QUESTION 10

(a) $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$

$1^3 + 2^3 + 3^3 + \ldots + (2n+1)^3 = \frac{(2n+1)^2(2n+1+1)^2}{4}$ ✓M — Sub. in $2n + 1$

$= \frac{(2n+1)^2(2n+2)^2}{4}$ ✓A

$= \frac{(2n+1)^2 2^2 (n+1)^2}{4}$ ✓M — Factor

$= (2n+1)^2(n+1)^2$ (3)

(b) $2^3 + 4^3 + 6^3 + \ldots + (2n)^3$

$= 2^3.1^3 + 2^3.2^3 + 2^3.3^3 + \ldots + 2^3.n^3$

$= 2^3(1^3 + 2^3 + 3^3 + \ldots + n^3)$ ✓M Factor

$= \dfrac{8.n^2(n+1)^2}{4}$ ✓A

$= 2n^2(n+1)^2$ ✓A (3)

(c) $1^3 + 3^3 + 5^3 + \ldots + (2n+1)^3$ ✓M ✓A

$= 1^3 + 2^3 + 3^3 + \ldots + (2n+1)^3 - \{2^3 + 4^3 + 6^3 + \ldots + (2n)^3\}$ Difference

$= (2n+1)^2 (n+1)^2 - 2n^2(n+1)^2$ ✓A

$= (n+1)^2 \{(2n+1)^2 - 2n^2\}$ Factorising

$= (n+1)^2 \{4n^2 + 4n + 1 - 2n^2\}$

$= (n+1)^2 (2n^2 + 4n + 1)$ (3)

9 marks

Total: 150 marks